UNIVERSITY OF ALBERTA

ON THE PROCUREMENT OF REACTIVE POWER SUPPORT SERVICES FROM GENERATORS

By

Yuanning Wang ©

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LIST OF SYMBOLS AND ABBREVIATIONS

t	Time instant
$p(t)$	Instantaneous power
$v(t)$	Instantaneous voltage
$i(t)$	Instantaneous current
V, I	RMS value of voltage and current
$\overline{V},\overline{I}$	RMS phasor voltage and current
θ	Phase angle difference between $\overline{V}$ and $\overline{I}$
ω	Frequency in rad/sec
P	Active power
Q *or var*	Reactive power
S	Apparent power
R	Resistance
X	Reactance
E	Sending end voltage unless defined otherwise
P_g, P_{out}, P_{output}	Active power output of a generator
Q_g, Q_{gen}, Q_{output}	Reactive power output of a generator
P_L, Q_L	Active power and reactive power load
Q_{sys}	Reactive power supplied from the system
Q_{losses}	Reactive power losses
$Q_{support}$	Reactive power support
$Q_{P\text{-}shipment}$	Reactive power used for transmitting active power
Q_{min}	Minimum reactive power need of a generator
pu, p.u.	Per unit quantities
APR	Abbreviation of active power reduction
OPF	Abbreviation of optimal power flow

Chapter 1

Introduction

The electric power industry is experiencing a lot of significant changes brought about by deregulation. In the past century, electric power systems have been organized and operated as regulated monopolies. It has been assumed that electricity and its delivery were inevitably intertwined. Power companies not only constructed and owned generating plants but also built and operated power transmission networks. The advancement in power generation technologies in the 1980s encouraged the building of smaller and environmentally friendlier generating units by non-utility companies, which made competition in power production possible. This economic reality has forced the power industry to break its century-old mentality and to define power transmission as a transportation service separate from the electric energy itself. Electric energy thus became a product that could be bought, sold and transported from place to place, much like gas or oil, in a competitive market. The intent of deregulation and restructuring is to push the electric power industry into a competitive environment in order to improve cost effectiveness and customer service satisfaction. This change has brought new technical challenges to the power engineering field. This thesis investigates one of the challenges emerging from the market structure.

1.1 Structure of the Deregulated Power Market

In the new deregulated environment, power systems are no longer controlled by single vertically integrated utilities. The new market consists of Generation companies, Transmission companies and Distribution companies. These entities must work independently and cooperatively to provide cost effective and reliable electric power supply. Another entity usually designated as the Independent System Operator (ISO) coordinates the activities of the above different functional parties to achieve the overall goal of serving the customers. A brief description of these entities is given as follows.

- The ISO is a neutral and independent operator responsible for maintaining instantaneous energy balance in the system. The ISO performs its function by controlling the dispatch of generation and gives orders to adjust or curtail loads to ensure that loads match available generating resources in the system. The general objective of the ISO is to guarantee a comparable and non-discriminatory open access to power suppliers and users of electric transmission systems. The ISO has the operational control of the transmission grid components, administers system wide transmission tariffs, maintains and ensures system reliability.

- A Generation company operates and maintains one or more generating plants. A Generation company may own generating plants or interact on behalf of plant owners with the power market. The Generation companies have the opportunity to sell electricity to entities with whom they have negotiated sales contracts. In addition to active power (electric energy), a Generation company may provide reactive power and operating reserves to make a profit.

- The Transmission system is a crucial element in an open electricity market. The secure and efficient operation of the transmission system is the key to efficiency in these markets. A Transmission company is composed of a high-voltage bulk electric power system that is shared by all participants. The Transmission companies are regulated to provide non-discriminatory connections and comparable service for their cost recovery. They provide wholesale transmission of electricity and offer open access to Generation companies.

- A distribution company is an entity that distributes electricity through its facilities to customers. It constructs and maintains distribution wires connecting the transmission grid to the customers. It also has the responsibility of responding to power quality concerns. In order to ensure the supply of power, Distribution companies coordinate their functions with the Transmission company and the ISO.

With the above entities involved, several open transmission access models have been proposed or practiced to facilitate competition [1, 2]:

- Poolco model: Under this model, one single entity, the Pool Company, purchases power from the competing generators in the open market and sells it at a single price to the distribution companies or to retail loads.
- Wholesale competition model: The retail loads can purchase power from one or more trading entities who are in competition with each other and who purchase their power from the competing generators.
- Retail competition model: This is the most general competition model. It allows for both of the above models as well as direct transactions between retail loads and generators.

At present, the Poolco model has gained wide acceptance in North America. Alberta is one of the provinces using a power pool scheme. Figure 1 shows the market structure of the power pool scheme. It has the following characteristics:

- All power producers can inject active power (energy) into the power pool.
- Electric energy and its delivery are no longer intertwined. Active power is traded as a commodity to be bought and sold.
- Various system support services are necessary to facilitate the transmission of the active power supplied from all sources. It is the system operator's responsibility to procure the various system support requirements.

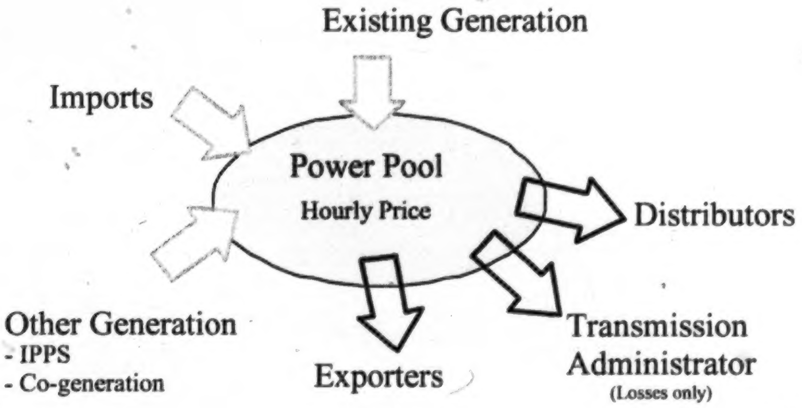


Figure 1.1: Structure of the deregulated power market

With such significant changes undergoing in the power market, one can envision lots of new technical tasks and challenges that need to be tackled to facilitate normal operation and real competition in the market.

1.2 System Support Services

For both reliability and commerce, bulk power systems require certain services to facilitate the secure and efficient transmission of electricity. These services are called ancillary services. The Federal Energy Regulatory Commission (FERC) of the United

States defined ancillary services as "those services necessary to support the transmission of electric power from seller to purchaser given the obligations of control areas and transmitting utilities within those control areas to maintain reliable operations of the interconnected transmission system" [3]. FERC identifies six ancillary services for open access transmission:

(1). Scheduling, System Control and Dispatch Service: This service requires the control-area operator to schedule generation and transactions. A control area must also dispatch generating resources to maintain generation/load balance and maintain security. This service is offered by an ISO with the time scale ranging from seconds to hours.

(2). Reactive Supply and Voltage Control: The control area operators provide this support service by controlling the reactive power level in the system to maintain voltages within a required range and to ensure that the system has enough voltage stability margin. This service should respond in seconds.

(3). Regulation and Frequency Response Service: This service uses generators equipped with automatic-generation control (AGC) to maintain a minute-to-minute generation/load balance within the control area to meet the scheduled interconnection frequency standards. The time scale of this service is about one minute.

(4). Energy Imbalance Service: This service uses generation to correct for hourly mismatches between actual and scheduled transactions between suppliers and the customers.

(5). Operating Reserve - Spinning Reserve Service: This service provides reserved generating capacity, which is synchronized to the grid and can respond immediately, to correct for generation/load imbalances caused by unplanned generation or transmission outages. The time scale of service is from seconds to 10 minutes.

(6). Operating Reserve - Supplemental Reserve Service: This service includes the provision of generating capacity and curtailable load used to correct for generation/load imbalances caused by generation or transmission outages. Supplemental operating reserve is provided by generating units that are on line but not supplying load. This service should be available within 10 minutes. Unlike spinning reserve, this supplemental reserve is not required to respond immediately.

These ancillary services are not new. Vertically integrated electric utilities have been performing these functions for decades as part of their normal operation. The costs to produce and manage these services have been included in the bundled electricity prices paid by retail customers.

The deregulated market now requires these services to be unbundled and priced separately. The ancillary services in the deregulated market have the following features:

- These services are not free.
- These services are not optional. They are essential for both reliability and commerce.
- Most of these services are provided by generators, the same pieces of equipment that produce the energy commodity.
- The traditional rules of thumb used to define the amount of each required service may not be suitable for competitive energy markets.
- These services need to be measured accurately and thus customers pay only for actual performance.

Competitive markets for ancillary services are both desirable and necessary. It thus becomes necessary to investigate the technical and economic issues related to the provision and competitive pricing of these services.

1.3 Reactive Power as An Ancillary Service

Among the above six key ancillary services, reactive power supply is an important service to maintain voltages throughout the transmission grid within the required limits. Reactive power also provides crucial support to enhance the system's power transfer capability. Reactive power from generators is of special importance since it is the main source of reactive power support and can respond to system constraints without delay. The reactive power support becomes particularly critical in competitive electricity markets with the entry of a large number of new players and the proliferation of power transactions.

Different from active power, reactive power has its own characteristics. It cannot be transported over a long distance, due to the large reactance of the power lines. As a result, reactive power support is usually provided locally. There are many reactive power sources in the power system, such as generators, synchronous condensers, capacitors and Static Var Compensators (SVCs) etc. Among the various reactive power sources, a generator is the most important one because of its fast response when a system is close to the stability limit. A generator is special since it provides both active power and reactive power according to the capability curve. There is, therefore, a limit on the maximum reactive power each generator can put out. Furthermore, as is shown later, a generator can be a reactive power provider or a consumer. These factors make the procurement of reactive power very complicated in the deregulated market.

Reactive power support is beginning to be treated as an ancillary service worldwide. In the UK, the National Grid Company, which carries out the functions of the ISO, invites tenders for reactive power support services. Generators can bid for reactive power support [4]. In the New York system, the ISO is responsible for procuring reactive power support and the service is provided at embedded cost-based prices. Generating resources, which operate within their capability limits, are directed by the ISO to produce or absorb reactive power to maintain voltages within limits. The ISO provides compensation to generators in case of revenue lost due to increased reactive power generation requests [5]. The Australian electricity market and its ISO also recognize reactive power as an ancillary service and financial compensation is provided to generators and synchronous compensators for their service provisions [6]. All reactive power support providers are eligible for their preparedness to provide the service when called for. These developments in the deregulated electricity markets indicate the trend toward treating reactive power as an ancillary service and creating financial compensation schemes for reactive power providers.

As an important service to the power market, reactive power support should be remunerated. Establishing an appropriate scheme to value and compensate reactive power support is important both operationally and financially for the new market. The existing pricing schemes, such as the fixed tariff for reactive power support suggested by FERC, however, are not sufficient to provide a technically sound and practical guideline for competitive reactive power procurement. Conventional practice requires generators to operate at a power factor within the range from 0.9 leading to 0.95 lagging. This regulation-based approach is not compatible with the competitive market. There is a general lack of understanding of the role expected from the reactive power support services. How to fairly monetarily compensate the reactive power support service remains an unsolved subject.

1.4 Objective and Scope

As introduced in the previous sections, power producers can sell active power as a commodity and/or sell reactive power as an ancillary service. In pursuit of profit, a power producer could face the eventuality of either producing more active power or producing more reactive power. Therefore it becomes a timely task to quantify the value of the reactive power support under the deregulated power market.

The objective of the research work in this thesis is to develop the concepts and schemes needed for equitable reactive power support valuation, which will form a technical basis for the competitive procurement of reactive power support services.

The scope of this thesis can be summarized as follows:

- Investigating the role of a generator's reactive power output. Since a generator produces both active and reactive power and can operate as either providing or consuming reactive power, it can be speculated that a generator may not always fulfill the reactive support role. It is, therefore, important to understand the physical characteristics and operational aspects of a generator and analyze the role a generator plays in the power market.

- Defining an appropriate index to assess the value of a generator's reactive power support. An appropriate index is necessary to quantitatively assess the value of a generator's reactive power support. This index represents the relative importance of each generator. It could provide a technical foundation to establish a price signal for a fair compensation scheme.

- Developing a method to determine the index and verifying both the index and method in large systems. The method should be effective and practical for both small and large systems, and should be tested on actual power systems. Such studies will help to verify the validity and performance of the method for real-life applications.

- Discussing the practical considerations and constraints of the proposed method. Testing the proposed method on large power systems could reveal the problems in practical use. The limitation and constraints of the proposed method will be discussed.

- Applying the proposed method to design reactive power support compensation schemes that can reflect the relative contribution of each generator. The schemes will foster the development of a truly competitive electricity market.

1.5 Outline of the Thesis

This research work tries to address the valuation problem of the reactive power support from generators. The contents of the thesis are organized as follows:

Chapter 2 provides an overview of reactive power in electric power systems. The characteristics of reactive power are first discussed. The sources of reactive power are introduced. The function of reactive power as a system support is investigated. The existing concepts and methods that are used in reactive power valuation related problems are reviewed.

Chapter 3 investigates the role of a generator's reactive power output using simple examples. It is illustrated that a generator's reactive power output has two components, and each component has its own functions. The two components are defined and the dual functions of a generator's reactive power output are demonstrated with study results. When the two components are successfully separated, the valuation problem of a generator's reactive power support can be readily solved. A method has been proposed to separate the two components.

Chapter 4 investigates the problem of reactive power support valuation from an alternative perspective. This chapter defines and analyzes a new concept — Minimum Reactive Power Requirement (Q_{min}) of a generator. The characteristics of Q_{min} are discussed. A practical method to determine Q_{min} in large power systems is also developed in this chapter. An illustrative example is given to show how the proposed method works. This method is then compared with the separation method in Chapter 3. It is concluded that this method is more appropriate for practical use.

Chapter 5 presents case study results using the method proposed in Chapter 4 in actual large power systems. Some practical considerations in implementing this method are discussed. Sensitivity studies are performed to evaluate the impact of different parameters on the Q_{min} value. These parameters include different active power output levels, plant locations, shunt compensations etc. It also discusses how to interpret the Q_{min} results in different scenarios.

Chapter 6 applies the Q_{min} results in two possible ways. One is to use Q_{min} results as weighting factors in transmission cost allocation. The other application is to design schemes for compensating generators that truly provide reactive power support to the system.

Chapter 7 describes one specific issue in calculating the Q_{min} results: the modeling of a generator's reactive power limit. This problem is encountered in many system studies. Different models to represent a generator's reactive power limit are evaluated. The impact of these models on load flow studies, voltage stability margins and convergence characteristics are discussed.

Chapter 8 presents a problem encountered at the early stage of this research: the ill-conditioning problem in load flow study. The relationship between ill-conditioning and voltage instability is investigated.

Chapter 9 provides the conclusions drawn from the work described in this thesis and gives suggestions for future research.

Appendix A provides brief geographic structures and maps of the BC Hydro and Alberta systems.

Appendix B documents the system data of some small test systems used in this thesis.

Chapter 2

Reactive Power Support Service in Power Systems

A basic requirement in the supply of electricity is to ensure that voltages are within a specified range at all buses. A power system is also required to maintain sufficient margin to prevent voltage instability. The satisfaction of these requirements relies on the adequate availability of reactive power support from all sources. It is therefore important to have a better understanding of reactive power support service in power systems. This chapter discusses in detail the nature and characteristics of reactive power support.

2.1 Reactive Power

When there is a sinusoidal voltage source and an inductive/capacitive device in the system, there will exist a lagging/leading sinusoidal current in the circuit. The instantaneous power is defined as the product of voltage and current at any instant t, as described in the following equation [7]:

$$p(t) = v(t) \bullet i(t) = VI\cos\theta\,(1+\cos 2\omega t) + VI\sin\theta\,\sin 2\omega t \tag{2.1}$$

where $v(t)$, $i(t)$ are expressed by $v(t)=\sqrt{2}V\cos\omega t, i(t)=\sqrt{2}I\cos(\omega t-\theta)$, respectively. V and I represent the RMS value of voltage and current. θ is the phase angle difference between voltage and current, and $\omega=2\pi f$ is the angular frequency. The second item in

Equation (2.1) is the instantaneous reactive power. It oscillates at a frequency of 2ω and has an average value of zero. The instantaneous reactive power represents the flow of energy alternately toward the load and away from the load. Reactive power is defined as the magnitude of the instantaneous reactive power. Reactive power consumed or generated by a device can be determined from the following equation:

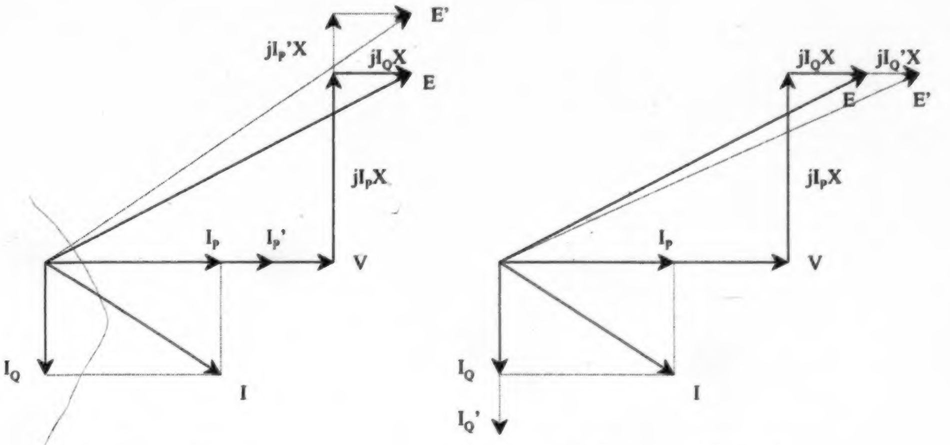
$$Q = \mathrm{Im}(\overline{S}) = \mathrm{Im}(\overline{V}\,\overline{I}^{*}) = VI\sin\theta \tag{2.2}$$

where $\overline{S}$ represents the apparent power and $\overline{V}, \overline{I}$ indicate rms phasors of voltage and current. Im (x) returns the imaginary part of x.

In power systems, most power producers generate both active power and reactive power. The transmission of active power and reactive power depends on the voltage magnitudes and angles. It is well known that active power and voltage angle are closely related, while reactive power and voltage magnitude are tightly coupled. This can be shown using phasor diagrams. Figure 2.1 represents the phasor diagram for a transmission line with reactance X. The resistance R of the transmission line is neglected for simplicity. The sending end voltage E and receiving end voltage V are calculated according to Equation (2.3):

$$\overline{E} = \overline{V} + j\overline{I}X = \overline{V} + jI_P X + jI_Q X \tag{2.3}$$

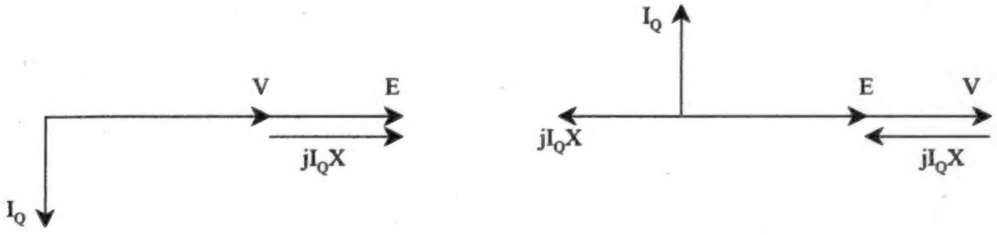
In Equation (2.3) line current I is separated into I_P and I_Q. The direction of the current I is from the sending end to the receiving end. When P increases, as shown in Figure 2.1(a), I_P will increase to I_P'. Voltage E will also increase to E'. Comparing E and E', it can be found that the phase angle difference between E and V increases more than the magnitude difference. Similarly, when Q increases, as shown in Figure 2.1(b), the change of magnitude difference is greater than the angle difference. Therefore, voltage angle is more related to active power and voltage magnitude is affected more by reactive power.



(a). Increase P load (b). Increase Q load

Figure 2.1: Impact of P and Q on voltage

The receiving end voltage of the transmission line can be higher or lower than that of the sending end, depending on the type of load connected. When the transmission line is supplying power to a capacitive load, which generates reactive power, the receiving end voltage will be higher than the sending end; On the other hand, when the transmission line is connecting to an inductive load, which absorbs reactive power, the receiving end will have a lower voltage. This is shown in Figure 2.2 by assuming I_P=0. E represents the sending end voltage and V is the receiving end voltage.



(a). Inductive load (b). Capacitive load

Figure 2.2: Sending and receiving end voltages of a transmission line

When electricity is transmitted through a transmission line, a current I flows in the transmission line. This current will cause both active power losses and reactive power losses on the line. Active power losses are associated with the resistive element (R) of the line and are given by I^2R, while reactive power losses are related to inductive reactance X and are defined as I^2X. For high voltage power lines, X is much greater than R, the reactive power losses are, therefore, considerably larger than the active power losses. Consequently, reactive power cannot be reasonably transported over long distances.

2.2 Sources of Reactive Power

In power systems, various types of equipment produce or absorb reactive power, as shown in Figure 2.3. Transformers, reactors and transmission lines are the main components consuming reactive power. Reactive power can be provided by a variety of devices. These devices can be categorized into two types: static and dynamic reactive power sources. Static reactive power sources, such as capacitors, cannot automatically respond to disturbances. They therefore show a slow response feature. Dynamic reactive power sources can respond rapidly to disturbances by automatically adjusting their reactive power output. Typical dynamic reactive power sources are generators, synchronous condensers, static condensers and some automatically switched capacitors. Some of these devices are described as follows.

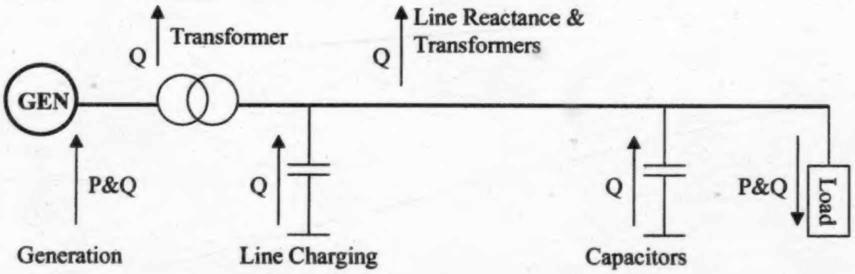


Figure 2.3: Illustration of reactive power flow

- Generators: Generators can produce both active power and reactive power. They can also generate or absorb reactive power depending on their excitation system settings. When overexcited they supply reactive power, and when underexcited they absorb reactive power. The capability to continuously supply or absorb reactive power is, however, limited by the field current, armature current, and end-region heating limits. Generators are normally equipped with automatic voltage regulators that continually adjust the excitation so as to control the reactive power output. There is a maximum limit on the reactive power that a generator can produce. When the limit is reached, the generator loses the ability of voltage control.

- Synchronous condensers: A synchronous condenser is a synchronous machine running without a prime mover or a mechanical load. Much like the generator, it can either generate or absorb reactive power by controlling the field excitation. Using a voltage regulator, it can automatically adjust the reactive power output to maintain constant terminal voltage. Despite the high purchase and operating costs of synchronous condensers, they have several advantages, such as instantaneous increase of reactive power output in case of voltage drop, tens of seconds of overload capability and excellent ability to cope with low system voltage conditions.

- Static Var Compensators (SVCs): SVCs are shunt-connected reactive power generators and/or absorbers whose outputs are varied so as to control specific operating parameters of the electric power system. Unlike synchronous condensers and generators, they have no moving or rotating components. SVCs may have different types of configurations. From the viewpoint of power system operation, however, a SVC is equivalent to a shunt capacitor and a shunt inductor, both of which can be adjusted to control voltage and reactive power at its terminals. Due to their ability to provide continuous and rapid control of reactive power and voltage, SVCs can enhance several aspects of transmission system performance, such as controlling temporary overvoltage and preventing voltage collapse. SVCs are very valuable when there is little generation in the load area. They have limited overload capability, but their capital costs are generally 20~30% lower than those of generators.

- Fixed or switchable shunt capacitors: As shown in Figure 2.2, shunt capacitors supply reactive power and boost local voltages. In general they provide the most economical reactive power sources for voltage control. Automatically switched shunt capacitors are used extensively in distribution systems to correct power factor and control feeder voltages. In transmission systems, shunt capacitors are used to compensate for reactive power losses and to ensure satisfactory voltage levels during heavy loading conditions. They are breaker-switched either manually or automatically by a voltage relay. The principal advantages of shunt capacitors are their low cost and their flexibility of installation and operation. The principal disadvantage is that their reactive power output is proportional to the square of the voltage. Consequently, the reactive power output is reduced at low voltages when it is likely to be needed most.

- Fixed series capacitors: They are connected in series with the line conductors to reduce the inductive reactance of the line. Series capacitors contribute to decreasing the overall reactive power losses, increasing the maximum power that can be transmitted and improving voltage profile. They are self-regulating, i.e., their reactive power outputs increase with line loading.

Among the above reactive power and voltage control devices, generators are probably the most important sources of dynamic reactive power. Since system operators have no direct control over the generators in a deregulated market, and the generators typically account for the largest portion of available dynamic reactive power, it is of special interest to investigate the value of a generator's reactive power output. Furthermore, generators usually produce active power and reactive power at the same time. They can also serve as either reactive power providers or consumers. These features make them important and unique players in the deregulated power market.

2.3 Reactive Power Support Service

Reactive power provides system support by two means: 1). Maintaining voltage profiles; 2). Enhancing power transfer capability. They are discussed respectively in this section.

2.3.1 Support for Voltage Profile

The effect of reactive power support on system voltage is illustrated using the system in Figure 2.4[1]. In Figure 2.4, a generator is supplying power to the load (P+jQ_L) through a transmission line with line reactance X. Local reactive power injection Q is available at the load site. The following equations apply:

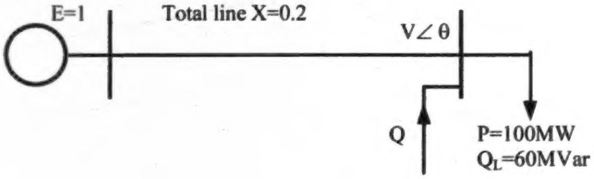


Figure 2.4: Sample system (Base: 100MVA)

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[1] Detailed data of this simplified system are shown in Appendix B. In this example, the line resistance is ignored for illustrative purpose. The line reactance X can be viewed as the equivalent Thevinin reactance of the system, where the line susceptance has been taken into account. This is applicable to other simplified small systems used in this thesis.

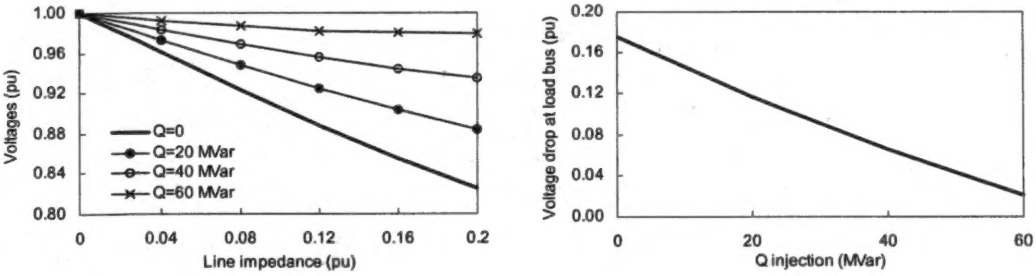
$$\begin{aligned} \overline{S} &= \overline{V}\,\overline{I}^{*} = P + j(Q_L - Q) \\ \overline{I} &= \frac{\overline{E} - \overline{V}}{jX} \end{aligned} \tag{2.4}$$

where *E* is the terminal voltage of the generator and *V* is the voltage at the load bus. Solving this equation, voltage *V* is obtained by:

$$V = \sqrt{\frac{1}{2}(E^2 + 2(Q - Q_L) \pm \sqrt{E^4 + 4(Q - Q_L)XE^2 - 4P^2X^2})} \tag{2.5}$$

Although Equation (2.5) gives two solutions for *V*, only one solution exists in real operation. The system can only operate at the voltage close to 1.0 per unit, where the '+' sign is used. Equation (2.5) shows that the load voltage *V* is closely related to the amount of reactive power injection. When more *Q* is injected, a higher load voltage is obtained. This is illustrated in Figure 2.5.

Figure 2.5(a) shows the voltage profile along the line. It is clear that the voltage decreases with distance from the generator (reactive power source). The voltage is the lowest when the load end is reached. When reactive power is injected at the load end, the voltages at all points along the line improve. This is because the reactive power injection directly cancels the reactive power load in this case, which leads to a smaller current. Consequently, voltage drop on the line becomes less. The more reactive power is injected, the higher the resulting voltage. Figure 2.5(b) shows that the voltage drop at the load end decreases when reactive power is injected. The injected reactive power is, actually a form of reactive power support to the transmission system. It helps to improve the voltage profile throughout the system. Figure 2.5 also shows that when the reactive power support is sufficient to cover the reactive load, i.e. the equivalent *Q* load is equal to zero, there is still voltage drop along the line. This voltage drop is due to the transmission of pure active power. In other words, the transmission of active power needs the support of reactive power [2].



(a). Impact of Q injection on voltage profile along a line

(b). Impact of Q injection on voltage drop

Figure 2.5: Impact of Q injection on voltage profile

2.3.2 Support for Power Transfer Capability

Using the same system in Figure 2.4, the effect of reactive power support on system power transfer capability is also illustrated. The power transfer capability describes the ability of a system to maintain acceptable voltages at all buses in the system under normal operating conditions, and after being subjected to a disturbance. The most common method to describe power transfer capability is the power versus voltage curve, or the so-called PV curve. Although other indices could also be used, years of industry experience have shown that the PV curve is the most effective tool to measure the capability of a power transmission system [8].

PV curve is usually obtained by increasing all the active and reactive loads of a system proportionally in steps. The active power generation is also scaled up to maintain power balance in the system. The voltages of selected load buses are then plotted with respect to the load level increase. Figure 2.6 shows a typical PV curve. The load characteristic curve can be modeled as a vertical line. The intersection of the PV curve and the load characteristic curve is the system operating point. In Figure 2.6, Point A is the operating point with load level P_l. Point B represents the maximum power transfer capability of the system, P_{max}. The difference between P_l and P_{max} is defined as the system margin. When there is a contingency, such as a line outage, the PV curve will shrink, P_{max} will decrease and the margin will become smaller. When the reduction of P_{max} is so significant that there is no intersection between the load curve and the PV curve, the system will experience voltage collapse. This is a progressive and uncontrollable drop in voltage at the buses.

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[2] Line susceptance, also known as line charging, also provides reactive power support to the system. The impact of line charging on system voltage can be significant. In such circumstances that the reactive power support from line charging reaches a certain level, voltage increase can be expected.

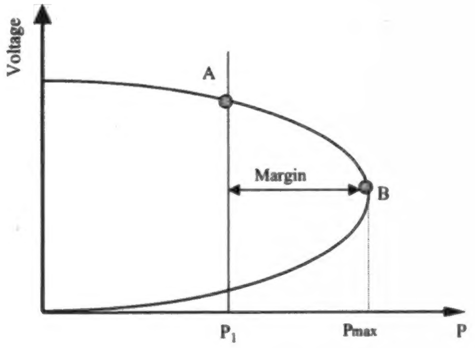
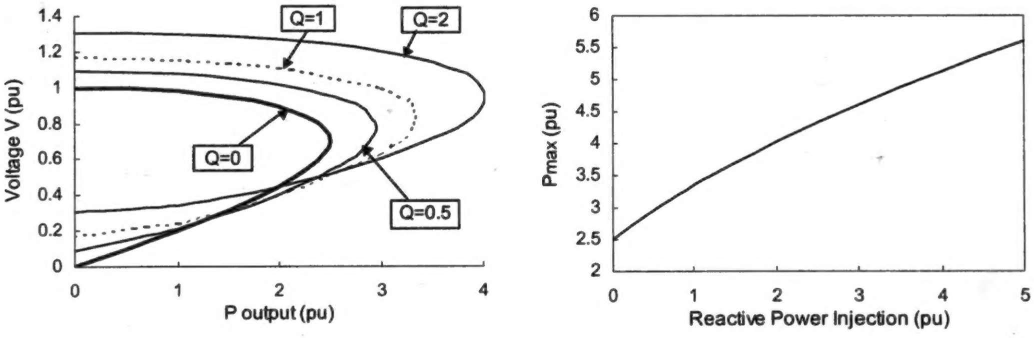


Figure 2.6: Sample PV curve

For the system in Figure 2.4, the voltage of the load bus is monitored. A family of PV curves is obtained when different amounts of Q are injected in the system, as shown in Figure 2.7(a). For simplicity, the reactive load Q_L is set to zero.

Figure 2.7(a) depicts the impact of reactive power support on the PV curve. It shows that the PV curve is extended when reactive power support is injected in the system. The maximum power transfer capability, represented by the furthest point of the PV curve, increases when more reactive power support is provided. Figure 2.7(b) shows the changes in maximum power transfer capability with respect to the changes in reactive power support. Figure 2.7 illustrates that reactive power support can greatly enhance the power transfer capability of a system.



(a). Impact of Q support on PV curve (b). Impact of Q support on P_{max}

Figure 2.7: Impact of reactive power support on power transfer capability

In summary, the impact of reactive power support can be considered as expanding the system capability curve in two directions, as shown in Figure 2.8. It supports the system voltage profile in the vertical direction and extends the network power transfer capability in the horizontal direction.

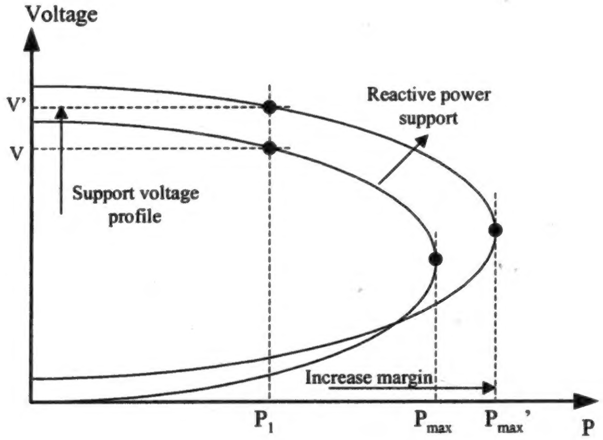


Figure 2.8: Impact of reactive power support

2.4 Literature Review

Due to the importance of the reactive power support services outlined in the previous section, research activities on methods for valuation or pricing of reactive power have become quite active in recent years. Many papers have been published in the area of optimal pricing of reactive power or the valuation of reactive power support services. These works can be classified into two broad groups as shown in Figure 2.9. One group deals with the problem using economic theory and the other is based on power engineering knowledge.

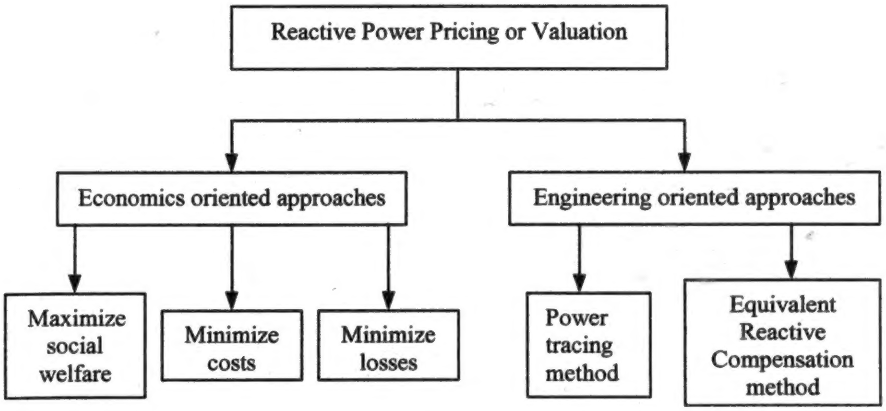

Reactive Power Pricing or Valuation
Economics oriented approaches
Engineering oriented approaches
Maximize social welfare
Minimize costs
Minimize losses
Power tracing method
Equivalent Reactive Compensation method

Figure 2.9: Classification of published papers on reactive power pricing and valuation

2.4.1 Economics Oriented Approaches

Research work in this category attempts to find an optimal price for reactive power to achieve certain economic goals. Such goals include maximum social welfare, minimum costs and minimum losses. The ideas are originated from the optimal spot price work developed for active power.

The spot price theory was first introduced into power systems by F.C. Schweppe, M.C. Caramanis and R.E. Bohn for active power in 1985 [9,10]. The theory states that there is a location and time dependent (i.e. spot-dependent) optimal price for supplying and consuming active power. The optimal spot price is derived when the global social welfare is maximized. It is shown in [9,10] that the optimal spot prices are equal to the marginal costs of providing electric energy to customers. The spot price for the k^{th} customer, $\rho_k(t)$, can be described as

$$\rho_k(t) = \frac{\partial}{\partial d_k(t)}[\text{Total cost of providing electric energy to all customers}] \tag{2.6}$$

where $d_k(t)$ is the load demand of customer k. Note that the price is a function of the customer location as well as time. The spot pricing theory is the application of basic economic theory to power markets. Schweppe saw spot pricing as a promising way to operate a competitive electricity market. His work provides the foundation and starting point for most subsequent research. One of these research works is the extension of the theory to the pricing of reactive power [11-16].

Different methods have been proposed to formulate and solve the problem of reactive power pricing, all under the framework of optimal pricing. The most commonly used objectives in the optimization are to achieve maximum social welfare [11], minimum costs [12-20] and minimum losses [21-23]. Baughman and Siddiqi analyzed real-time pricing of reactive power using a modification of the Optimal Power Flow (OPF) model [12]. The new model is a bi-level problem consisting of two parts. The upper level problem is to satisfy the demand function, which allows for price responsiveness of active and reactive power demand. The lower level problem is the optimal power flow problem, which has the objective of minimizing the total cost of operating the generating units subject to system and operational constraints. Similar to the definition of price for active power in Equation (2.6), the price of reactive power is also found as the marginal cost to satisfy reactive power demand. This model reveals that the Lagrangian multipliers represent the marginal costs of node power injections and shows OPF can be a promising tool for spot pricing. Baughman and Siddiqi later extended their model to incorporate constraints on power quality and environment impact that often influence the operation of a power system [11].

Many other papers also apply the well-known marginal price theory of economics to determine optimal prices for reactive power. The differences among the various work are mainly in the formulation of the optimization problem. Reference [13] aimed at developing a price structure for providing reactive power service. Specifically, the economic and technical issues of providing reactive power services are discussed. Reference [14] presents an integrated framework to analyze the issues of reactive power planning along with reactive power pricing. The OPF objective function is modified to comprise the aggregate cost of generation and the cost of adding new capacitors. Furthermore, a two-part reactive power spot-pricing scheme is formulated to include a fixed part to cover the capital expenditure on new capacitors and a variable part to cover the operating costs. The variable part is similar to the scheme proposed in [12].

The concept of wheeling rate is proposed in [15] and [16]. Wheeling is defined as the transmission of active and reactive power from a seller to a buyer using a transmission network belonging to a third party. Wheeling rates are the prices the third party charges for use of its network. The rates are based on marginal cost pricing implemented using a modification of the OPF algorithms. The main objective of OPF is to find the output of each generator that minimizes the total operating costs.

With increasing concerns on reactive power pricing, some scholars focused on establishing an appropriate price structure for reactive power support. J. Lamont and J. Fu [17] analyzed the explicit and implicit costs of reactive support from generation and transmission sources. They proposed the concept of opportunity cost, which represents the value that cannot be achieved by producing reactive power. In this paper, a cost-based reactive power dispatch is presented, which minimizes the total cost of reactive power support.

Using the above pricing theory for reactive power, research work has also been conducted on designing the ancillary services markets for procurement of reactive power [18-22]. A market for ancillary services is designed in combination with a spot market for electricity in [18]. The developed algorithm uses a contingency based modeling approach. It selects reactive power offers so as to optimize power system security for least overall cost. Reference [20] presents a method for the simulation and analysis of the reactive power market. The value of reactive power support, in terms of both capability and utilization of each particular generator is quantified using a modified security constrained reactive optimal power flow. More recently, comprehensive discussions were presented in [21-23] to address the problem of reactive power procurement by an independent system operator (ISO) in a deregulated market. In these papers, a reactive bidding structure is proposed first in the context of a reactive power market. Based on the reactive power price offers and technical constraints involved in reactive power planning, a two-tier approach is developed to determine the most beneficial reactive power contracts for the ISO. The ISO first determines the marginal benefit of each reactive bid with the objective of seeking to minimize active power losses. With the marginal benefit known, the ISO then seeks to maximize a societal advantage function formulated by incorporating the price bid offers at this stage.

The published papers described above are oriented using economic theories and rules. While the spot price theory may well be applicable to active power, its adaptation to reactive power needs a lot of careful thinking. This type of work is summarized and discussed as follows:

- Derived from economic theories, the spot pricing theory is mathematically elegant. It is conceptually simple and easy to understand, though complicated in implementation. The theory establishes a theoretical justification for the need for location and time dependent prices.

- The spot pricing theory was originally developed for active power. Since active power can be sold and bought as a commodity, it is quite reasonable to determine the optimal price for the purpose of maximizing global social welfare or minimizing total costs. However, it is questionable to extend this theory to reactive power, which is a form of service. For example, how does one define a customer's reactive power demand when Equation (2.6) is applied to reactive power pricing? Note that the customers can have zero reactive power demand at their service entrance points and yet the system still needs reactive power support. Furthermore, since reactive power cannot be transmitted over long distances, it may not be possible to price reactive power from a system wide perspective as can be done to active power.

- It is also questionable to determine the optimal reactive power price using the marginal cost concept. This is due to two reasons. Firstly, the operational costs are mostly fuel-related. As a result, the marginal cost of reactive power, which represents the sensitivity of the generation production cost to the reactive power demand, is negligible compared to that of active power [13, 15]. Secondly, satisfying reactive power demands is only a very small function of reactive power support. For example, reactive power losses in the transmission network often exceed the total reactive power load. As a result, a customer demand based pricing could be misleading.

- Since the main role of reactive power is to support system voltage and margin, the problem should be formulated with this role as the primary objective. Otherwise, an optimal reactive price could lead to systems operating at poor stability level. One may argue that the optimal price theory could take into account these concerns by including more (stability-related) constraints to the optimization problem. They appear more like bandage solutions that have misplaced the main issues of the problem.

- The methodology of optimization tends to limit researchers to the input and output of the optimal power flow solutions, making it difficult to gain in-depth and intuitive understanding of the issues involved. The research on the problem of reactive support procurement is still in the early stage, there is, therefore, a need to understand the technical aspect of the problem better before one applies an 'all-encompassing' tool to solve the problem.

Finally, the spot price theory seems to lead to a regulated market for active and reactive power, i.e. a central agency (the optimization software) establishes power prices for the market. It appears to contradict with the goal of power market deregulation. At least, it is known that many real active power markets establish power prices based on competitive bidding instead of spot price theory.

2.4.2 Engineering Oriented Approaches

Another strategy to investigate the reactive power pricing problem is to approach it from the power engineering perspective. Indeed, there are at least two research directions published in literature using this strategy. One is the power tracing method [24-30] and the other is the equivalent reactive compensation (ERC) method [31, 32]. These methods are discussed in this section.

1). Power Tracing Method:

This method computes the power flowing from a given generator to each load or calculates each generator's contribution to a particular load. If such 'usage allocation' questions are answered clearly and unequivocally, it will be very useful to ensure the competitive market to be fair and efficient.

Two of the most popular types of power tracing method were developed by D. Kirschen and J. Bialek, respectively. They proposed generation distribution factors to determine the share of a particular generator in the line flow. Kirschen based his method on generator domains and started with tracing active power flows from generators to loads [24,25]. This method was extended to reactive power in [26]. Based on a solved power flow solution, all power injections are translated into real and imaginary currents to avoid the problems arising from the non-linear coupling between active and reactive power flows caused by losses. The method then traces these currents to determine how much current each source supplies to each sink. These current contributions can then be translated into contributions to the active and reactive power output of the generators. Bialek introduced a topological method to allocate the supplementary transmission charge to active and reactive loads [27-29]. Another method for determining the share of the generators in a customer load has been developed based on the nodal generation distribution factors [30]. It is stated that this method can be used for both active and reactive power flows since the transmission losses are taken into account.

Although the power tracing method emphasizes the technical aspect of reactive power procurement and brings up a potential research direction, the published methods all have some disadvantages. For example, Reference [26] uses active and reactive current. The methodology seems to be interesting and mathematically correct. However, it has no physical meaning and can be difficult to implement in practice. Reference [29] has errors in treating reactive power flows by analyzing active and reactive power flows independently. In general, these methods are more appropriate for active power than for reactive power. Furthermore, some of them lack technical foundation and sometimes are based on untenable assumptions [26].

2). Equivalent Reactive Compensation (ERC) Method [31, 32]

This method emphasizes the characteristics of reactive power and tries to analyze the reactive power valuation problem from the perspective of system security. The basic concept of the ERC method is to observe the system response when the reactive power output of the study source is varied from its minimum to maximum values. The technique can be understood as follows: If a var source changes its output, the network voltage profiles and security levels will change. To maintain the same degree of network security, additional reactive power support must be added. The total amount of reactive power support added is, in effect, a direct measure of the value of the missing var output from the study source. The added reactive power support is called the 'Equivalent Reactive Compensation (ERC)', since it's effect is equivalent to the var output from the study source.

The ERC method has a clear physical meaning and is practically applicable. However, subsequent work on the method reveals that it cannot distinguish two different functions of the reactive support from generators [33]. This is discussed further in the next chapter. In fact, the need to solve this problem leads to the research subject of this thesis.

Although the engineering oriented methods also need a lot of improvements, it appears that this research methodology is more suitable at this stage of the research work. The results of this approach will help people to gain more insights into the problems involved. It will be very difficult to develop an adequate solution if the problem is not fully understood.

How to competitively procure reactive power support service is also affected by factors other than economic and technical considerations. For example, it is equally important that a price scheme can be easily understood by customers and appreciated by regulators. An engineering oriented approach is also preferable from this perspective, since it is easier to explain the justifications behind the pricing proposals developed using engineering-oriented approaches. The research on reactive power support procurement is still in its infancy, and therefore establishing a technically sound understanding of the problem is much more important than applying sophisticated mathematical tools at the present time. Therefore, this thesis proposes to study and analyze the characteristics and functions of reactive power support service first in order to provide adequate technical basis for the valuation problem of reactive power services.

2.5 Summary

This chapter discusses the nature and characteristics of reactive power. It illustrates that reactive power support services have the following characteristics:

- Reactive power is critical in maintaining voltage profile. Without sufficient reactive power, it is impossible to maintain acceptable bus voltages.

- Reactive power is critical in maintaining adequate power transfer capability. Without sufficient reactive power, the system is unable to meet the load demand and can experience voltage collapse.

- There are sources and sinks of reactive power. Some devices such as shunt capacitors provide reactive power to the system. These are reactive power sources. On the other hand, some devices such as reactors absorb reactive power from the system. These are reactive power sinks.

- Reactive power can not be transported over a long distance. Power transmission lines usually have inductances much larger than the resistances. When there is current flowing in the transmission lines due to any energy transmission, these lines can incur large reactive power losses and voltage drops[3]. Therefore it is desirable to provide reactive power support locally.

- Generators account for the most reactive power sources in the power system. They can produce active power and reactive power at the same time. Generators can serve as either reactive power providers or consumers. Therefore, generators are an important dynamic source of reactive power.

The importance of reactive power support has led to a lot of research activities on the methods for valuation or pricing of reactive power. Both economics and engineering oriented approaches have been proposed by various researchers. The pros and cons of both approaches are discussed in this chapter. This thesis develops an engineering oriented approach to provide the necessary technical foundations for the valuation of reactive power services.

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[3] This refers to heavy load conditions. Under light load conditions when the load is smaller than the surge impedance loading, high voltage transmission lines generate surplus reactive power and therefore can cause voltage increase. This thesis analyzes systems under heavy load conditions.

Chapter 3

Dual Functions of A Generator's Reactive Power Output

The reactive power output of a generator has been viewed as providing voltage support to the grid for a long time. However, it is found in this thesis that a generator's reactive power is not always used merely for the support service. A generator also needs reactive power to transmit its generated active power. When there is a shortage of reactive power, the generator becomes incapable of transmitting its active power to the system. This phenomenon implies that providing system support is not the sole purpose of a generator's reactive power output. This chapter investigates this phenomenon in detail. The dual functions of a generator's output are examined and a method is proposed to separate the two functions.

3.1 Significance of Reactive Power Losses

When a generator sells electric power to the grid, there is a current flowing along the transmission lines. This current will result in reactive power losses, as explained in Chapter 2. If the generator does not output sufficient reactive power to cover the losses, the system has to provide reactive power back to support the generator's active power transaction activity. Therefore, it is conceivable that a generator could need more reactive power for its own active power transmission than the amount it produces. Consequently, the generator could 'tax' the system for reactive power and degrade the system security level instead of providing support to the system. This situation can be illustrated using a hypothetical example shown in Figure 3.1. Q_g in this figure represents the reactive power output of a generator and Q_{sys} is the reactive power received by the grid.

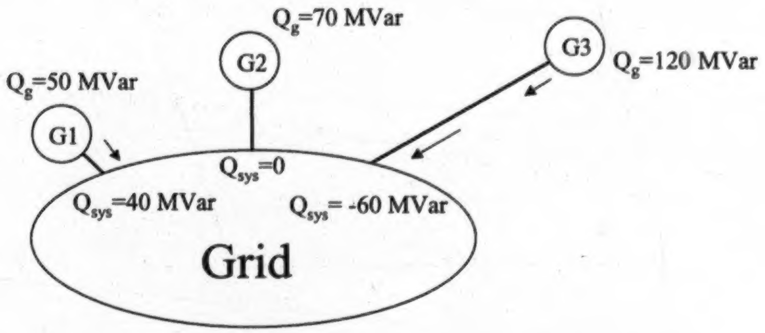

Figure 3.1: Role of a generator's reactive power output

The three generators in Figure 3.1 have different impacts on the system from the perspective of providing reactive power support:

- G1: although outputs a small amount of reactive power, it actually provides reactive power support to the system. Usually this type of generator is close to load center. Therefore, G1 plays a support role;
- G2: although outputs some reactive power, it neither provides support to the system nor absorbs support from the system. Its reactive power is used to cover the reactive power losses on the line. Thus G2 plays a neutral role;
- G3: although outputs a large amount of reactive power, it actually draws reactive power from the system. This is because its reactive power output is depleted by the long transmission line before reaching the load center. The generator needs the system to supply reactive power to facilitate its active power transaction. As a result, G3 cannot fulfill the support role.

Figure 3.1 shows that when a generator outputs a certain amount of reactive power to the system, it is possible that only a portion of the reactive power generated is available to support the system (such as G1 in Figure 3.1). It is also possible that the generated reactive power is not enough to transmit its own active power (such as G3). This shows that a generator does not always play a support role even if it produces reactive power. It is important to investigate this phenomenon. If this issue is not addressed properly, a remote generator with low fuel cost may have unfair advantages over an expensive generator close to the load center.

3.2 Dual Functions of A Generator's Reactive Power Output

The discussions in the previous section can be formulated more rigorously with the concept of dual functions. Dual functions mean that a generator's reactive power output has two functions, one is to support transmission of its own active power to the grid, and the other is to provide system support. The simple two-bus system in Figure 3.2 is used to illustrate the phenomenon. In this system, a generator supplies power to an equivalent load. For simplicity, the line resistance has been ignored[4]. The main concepts related to the problem are introduced using this system.

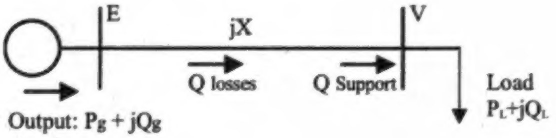

Figure 3.2: Two-bus simple system

For this simplified system, there exists:

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[4] Please refer to Footnote 1.

$$Q_g = Q_{losses} + Q_{support} \tag{3.1}$$

where Q_{losses} indicates all the reactive power losses on the branch and $Q_{support}$ represents the generator's reactive support to the system. Using I to indicate the branch current, there exist the following equations:

$$\begin{aligned} Q_{support} &= Q_L \\ Q_{losses} &= I^2X = \frac{P_g^2}{E^2}X + \frac{Q_g^2}{E^2}X \\ Q_g &= Q_L + \frac{P_g^2}{E^2}X + \frac{Q_g^2}{E^2}X \end{aligned} \tag{3.2}$$

Equation (3.2) can also be expressed by using V, P_L and Q_L to represent the branch current, as shown in Equation (3.3):

$$\begin{aligned} Q_{losses} &= I^2X = \frac{P_L^2}{V^2}X + \frac{Q_L^2}{V^2}X \\ Q_g &= \frac{P_L^2}{V^2}X + Q_L + \frac{Q_L^2}{V^2}X \end{aligned} \tag{3.3}$$

In this particular system, P_L=P_g since there is no active power loss. Therefore, Equation (3.3) is equivalent to

$$Q_g = \underbrace{\frac{P_g^2}{V^2}X}_{(1)} + \underbrace{Q_L + \frac{Q_L^2}{V^2}X}_{(2)} \tag{3.4}$$

It can be seen from Equation (3.4) that the generator's reactive power output has two components: Component (1) deals with the transmission of its own active power output P_g, while Component (2) represents the generator's reactive support to the system. Hence, the two components can be defined as follows:

$$\begin{aligned} &Component\_1: Q_{P-shipment} = Q_g \big|_{Q_{support}=0(Q_L=0)} = \frac{P_g^2}{V^2} X \\ &Component\_2: Q_{support} = Q_g \big|_{Q_{P-shipmen}=0(P_g=0)} = Q_L + \frac{Q_L^2}{V^2} X \end{aligned} \tag{3.5}$$

The above definitions show that $Q_{support}$ deals with reactive load Q_L, therefore, the function of $Q_{support}$ is to provide reactive power support to the system and maintain system voltage by satisfying the demand of reactive loads. On the other hand, $Q_{P\text{-}shipment}$ is related to the generator's active power output and the function of $Q_{P\text{-}shipment}$ is to compensate the reactive power losses caused by active power transmission.

3.3 Analysis of the Dual Functions

One of the possible methods to quantify the value of reactive power support service from a generator is to separate the two components of the generator's reactive power output for a given generation pattern. Such a method is proposed in Section 3.4. Before proceeding to the general method, it is useful to analyze the problem in more detail.

The three-bus system shown in Figure 3.3 is used for this analysis[5]. The system consists of two generators supplying one load. The reactive power support component $Q_{support}$ can be obtained by monitoring the reactive power flowing into bus 3 from the two branches. The component of $Q_{P\text{-}shipment}$ is the difference between the reactive power produced by the generator and that delivered to the load center. When the system operating parameters are changed, the values of these two components will change. The characteristics of a generator's reactive power output can be revealed by observing the variation of these values.

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[5] Detailed data of this system are shown in Appendix B.

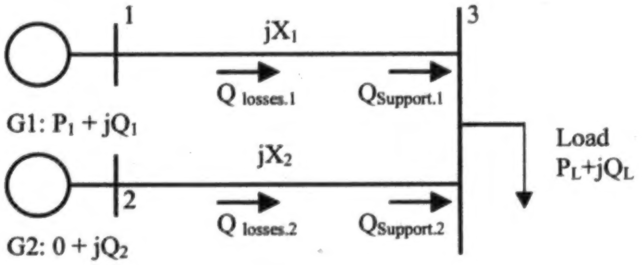


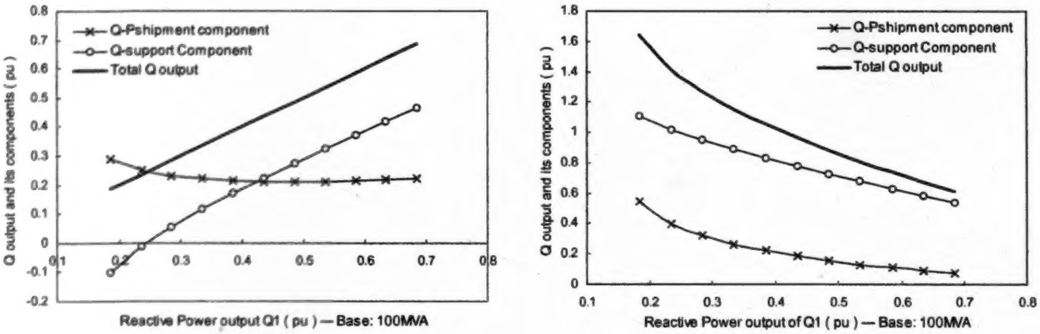
Figure 3.3: Three-bus system

For simplicity, the active power output of G2 is set to zero and the electrical distance of the two generators to the load is the same to avoid the impact of this factor. The study was conducted using the following two cases:

1). Varying reactive power output of G1:

The reactive power output of G1 ($Q1$) can be changed by varying its terminal voltage. When the terminal voltage is reduced to a certain value, the generator starts to absorb reactive power from the system. The system reactive power is actually supplied from G2. The impact of a $Q1$ change is shown in Figure 3.4(a) and 3.4(b), respectively.

Figure 3.4 shows the variation of the total reactive power output and its two components of G1 and G2 when the reactive power output of G1 changes. It can be seen from Figure 3.4(a), that when the reactive power output of G1 is reduced, the component of $Q_{support}$ also decreases, while the component of $Q_{P\text{-}shipment}$ increases. It can also be noticed that the lower part of the $Q_{support}$ curve is below zero. This indicates that when the reactive power output decreases to a certain level, generator G1 can no longer provide reactive support to the system. It actually draws reactive power support from G2 to transmit its own active power. In this case, although G1 generates reactive power, it actually has no capability to support the system. The generator G1, therefore, should pay a penalty instead of being compensated.



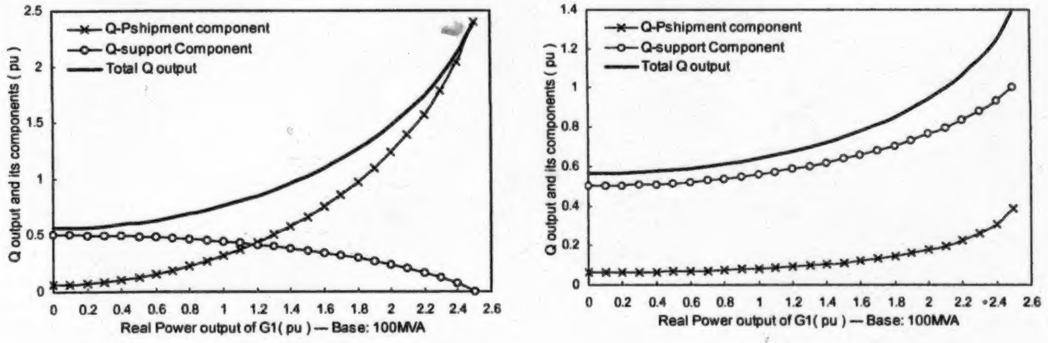
(a). G1 (b). G2

Figure 3.4: Variation of Q output and its components w.r.t. Q changes

2). Varying active power output of G1:

When the active power output is decreased, it can be expected that the reactive power component for active power transmission will decrease at the same time. Varying the active power output of G1, this phenomenon can be observed, as shown in Figure 3.5.

Figure 3.5(a) shows that generator G1 needs to output more reactive power in order to transmit more active power to the system. Most of the reactive power output of G1 is consumed in the increasing $Q_{P\text{-}shipment}$ component. As a result, less reactive power support is provided to the system.



(a). G1 (b). G2

Figure 3.5: Variation of Q output and its components w.r.t. P changes

The three-bus system shows the dual functions of a generator's reactive power output. When a generator's reactive power output decreases, it gradually loses the capability of providing system support and in the worst case, even extracts reactive power from the system to support its own active power transmission. When the active power output of a generator is reduced, the required reactive power for transmitting the active power also decreases.

3.4 A Method to Separate the Two Components of A Generator's Reactive Power Output

One approach to assess the true contribution of a generator's reactive power output is, therefore, to separate it into two components. One component is associated with the transmission of the generator's active power ($Q_{p\text{-}shipment}$) and the other component is to provide reactive support to the system ($Q_{support}$). The problem of separating the two components is designated as the reactive power separation problem. The separation method described in the previous section is case-specific. It is impossible to measure the support portion directly for general cases where several load centers exist. This section sheds light on the complexity of the separation problem for actual power systems. A practical separation method for solving the problem is then introduced.

3.4.1 Complexity of the Separation Problem

Many factors increase the complexity of the separation problem in actual power systems. The two-bus system of Figure 3.2 is used to give an example of the difficulties to solve the problem. For a given load demand and generator voltage setting, the reactive power output of the generator can be computed by solving the equations in (3.2). The solution for these equations is:

$$Q_g = \frac{E^2}{2X} - \frac{1}{2X}\sqrt{E^4 - 2XE^2Q_L^2 - 4X^2P_g^2} \tag{3.6}$$

It shows that the reactive output of the generator is a non-linear function of E, P_g, and Q_L. Even if this function could be separated into three decoupled functions, i.e., Q=A(E)+B(P_g)+C(Q_L), the separation problem would not be totally solved. The reason is that although the component B(P_g) is only related with P transmission, and the component C(Q_L) is totally related with reactive power support, the component A(E) is dependent on both variables P_g and Q_L. This illustrates the complexity of the separation problem. Furthermore, in actual power systems all the above functions are coupled. For instance, if the voltage setting is modified, the amount of reactive power for transmitting P_g is also modified. Therefore, it is necessary to develop approximate methods to separate the components of a generator's reactive power output with acceptable accuracy.

3.4.2 Active Power Reduction Method

One possible method to solve the separation problem is the Active Power Reduction (APR) method. This method is developed to obtain the remaining reactive power output of a generator when its active power output is reduced to zero. The necessary justification for this method is: 'If a generator outputs zero active power, its whole reactive power output is given for system support'. Using this point as a reference, an approximated separation for the other points is possible. Figure 3.6 is used to illustrate the procedure of this method. It is carried out as follows:

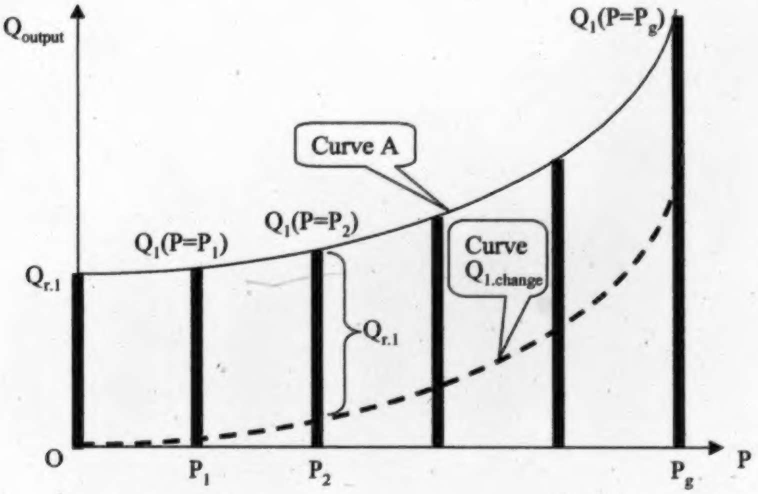


Figure 3.6: Illustration of the APR method

1). Select the generator whose reactive power is to be separated. This generator is called the test generator.

2). Decrease the active power output of the test generator to zero. Shed load accordingly. The purpose is to find a reference point, where the test generator outputs zero active power and its whole reactive power output is given for system support.

3). Solve load flow, record the reactive power output of all generators: $Q_{r.1}$, $Q_{r.2}$, ..., $Q_{r.j}$,..., $Q_{r.k}$, where j is the j^{th} generator and k is the number of generators. These values provide reference points for the generators, respectively. As shown in Figure 3.6, $Q_{r.1}$ is the reference point for Generator 1.

4). Increase the active power output of the test generator to P_1. Change load accordingly.

5). Solve load flow and record the reactive power output of all generators, $Q_1(P=P_1)$, $Q_2(P=P_1)$,…$Q_k(P=P_1)$. The situation of $Q_1(P=P_1)$ is shown in Figure 3.6. These values show how the generators respond to active power changes of the test generator.

6). Further increase the active power output of the test generator, repeat step 4) and 5) until the active power output of the test generator reaches the original base case value. Record the reactive power output of all generators at this point: $Q_1(P=P_g)$, $Q_2(P=P_g)$,…$Q_k(P=P_g)$.

7). Connect the recorded points with respect to different P values to obtain $Q_j \sim P$ curve for each generator, respectively. Curve A in Figure 3.6 is such a curve for Generator 1.

8). For each generator, determine the changes of reactive power output when P of the test generator varies. Using Generator 1 as an example, this is done as follows:

$$
\begin{aligned}
&Q_{1.change}(P=P_1)=Q_1(P=P_1)-Q_{r.1},\\
&Q_{1.change}(P=P_2)=Q_1(P=P_2)-Q_{r.1},\\
&\dots\\
&Q_{1.change}(P=P_g)=Q_1(P=P_g)-Q_{r.1},
\end{aligned}
\tag{3.7}
$$

The above procedure is equivalent to shifting Curve A down by $Q_{r.1}$, shown as Curve $Q_{1.change}$ in Figure 3.6. Shifting $Q_j \sim P$ curve of the j^{th} generator by $Q_{r.j}$,

$Q_{j.change}$ curve can be obtained for the j^{th} generator. A family of curves is then available by repeating this procedure for all generators. These curves represent the reactive power needed from each generator to support the active power changes of the test generator.

9). Superpose the $Q_{j.change}$ curves of all generators. This provides the $Q_{P\text{-}shipment}$ component of the test generator:

$$Q_{P\text{-}shipment}(P_i) = \sum_{j=1}^{k} Q_{j.change}(P_i) \tag{3.8}$$

where k is the total number of generators. P_i is the active power output of the test generator.

10). Once the component $Q_{P\text{-}shipment}$ is available, the component for reactive power support can thus be obtained for the test generator:

$$Q_{support}(P_i) = Q_{output}(P_i) - Q_{P\text{-}shipment}(P_i) \tag{3.9}$$

11). Correlate the calculated $Q_{P\text{-}shipment}$ (P_i) and $Q_{support}$ (P_i) with the P_i value respectively, the $Q_{P\text{-}shipment} \sim P$ curve and $Q_{output} \sim P$ curve of the test generator can be obtained.

3.4.3 Case Study Results

The APR method is illustrated using the five-bus system in Figure 3.7. This system consists of three generators supplying a load center (bus 5) that can be viewed as a power pool. A distant slack bus is used to provide angle reference. It contributes little to the system. When the P of the test generator (G_k) is reduced, all generators show a decrease in reactive power output. It means that the variations of all generators' Q output (Q_{1k}, Q_{2k}, Q_{3k}) are related to the P transmission of the test generator. The summation of these factors for any *P* point gives the total $Q_{P\text{-}shipment}$ for the test generator, which can be decreased from the test generator's total *Q* output, resulting in the $Q_{support}$ of the test generator.

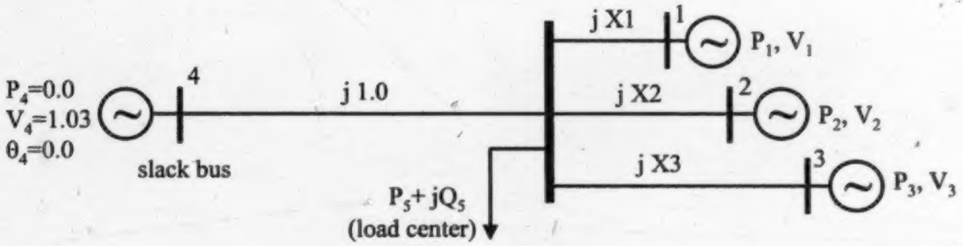


Figure 3.7: Five-bus sample system [6]

(Base: 100MVA)

Figure 3.8 illustrates the results of the APR method by testing G1 for the case where just the distances of the generators from the load center are different. G1 is the closest generator (*X1*=0.1pu) and G3 is the most distant one (*X3*=0.3pu, *X2*=0.2pu). Figure 3.8(a) represents the $Q_{output} \sim P$ curves of the three generators. When the active power *P* of G1 is reduced, all the three generators decrease their reactive power output. Figure 3.8(b) indicates the reactive power output changes of the three generators due to the variation of active power in G1. The curve 'Total *Q* Changes' is the sum of these three curves, which represents the necessary component for active power transmission. Figure 3.8(c) shows the *Q* output of G1 and its two components. It is observed that when active power *P* of G1 increases, the component for *P* transmission increases, while the component for *Q* support decreases.

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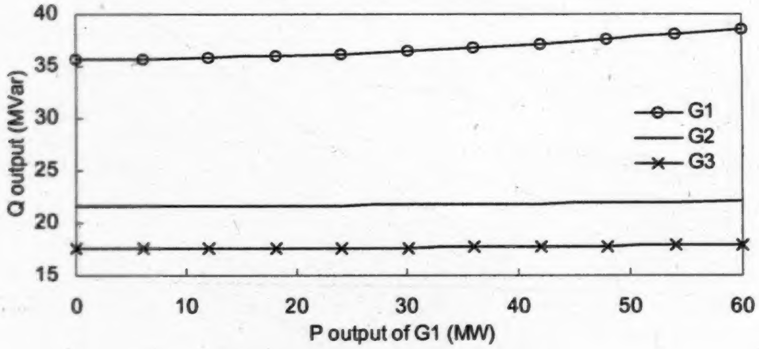
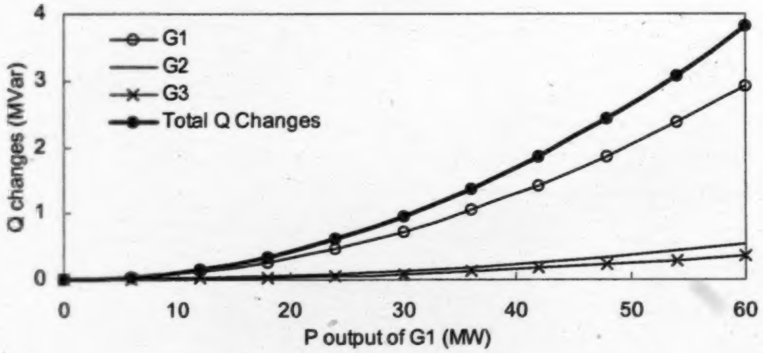
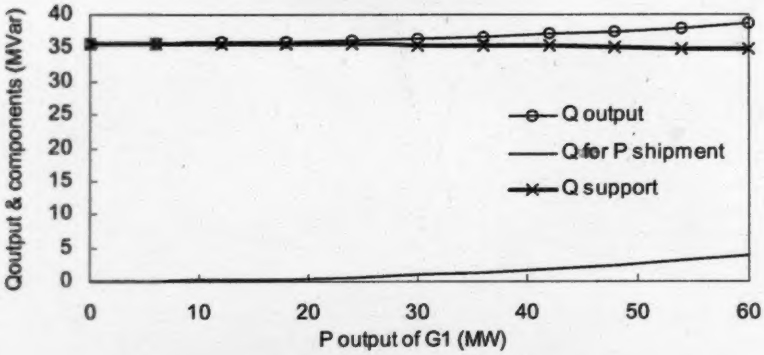
[6] Detailed data of this system are shown in Appendix B.



(a). Q output of three generators



(b). Q output changes



(c). Separation of $Q_{P\text{-}shipment}$ and $Q_{support}$ for G1

Figure 3.8: Separation of Q output components for test generator G1

Similar calculations are conducted when G2 and G3 are test generators respectively. The results, two components of each generator, are shown in Figure 3.9 and Figure 3.10.

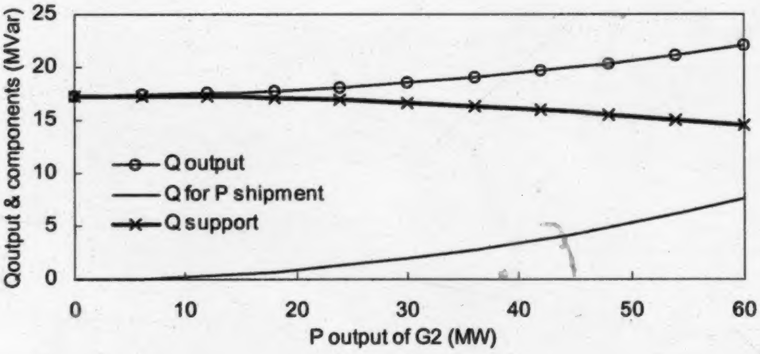


Figure 3.9: Separation of Q output components for test generator G2

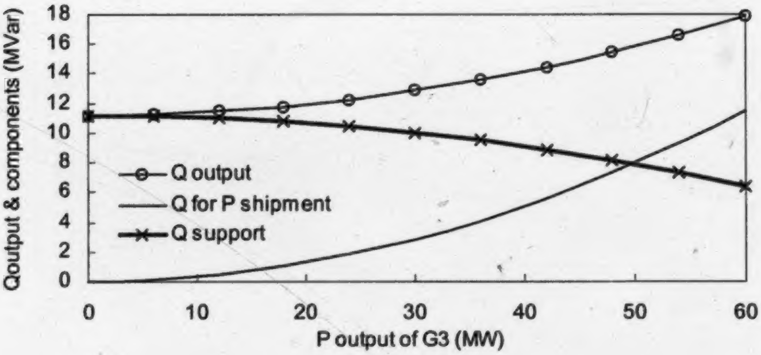


Figure 3.10: Separation of Q output components for test generator G3

The $Q_{support}$ components of the three generators are summarized and compared in Figure 3.11. It shows that $Q_{support}$ of the three generators all decrease when more active power P is transmitted. For a certain P output level, G1 provides the highest reactive power support since it is the closest generator. On the other hand, G3 provides the least reactive power support to the system due to its distant location. When active power output exceeds 140MW, G3 can no longer provide reactive power support. It will need reactive power from the system to support its active power transmission. These are consistent with engineering experience.

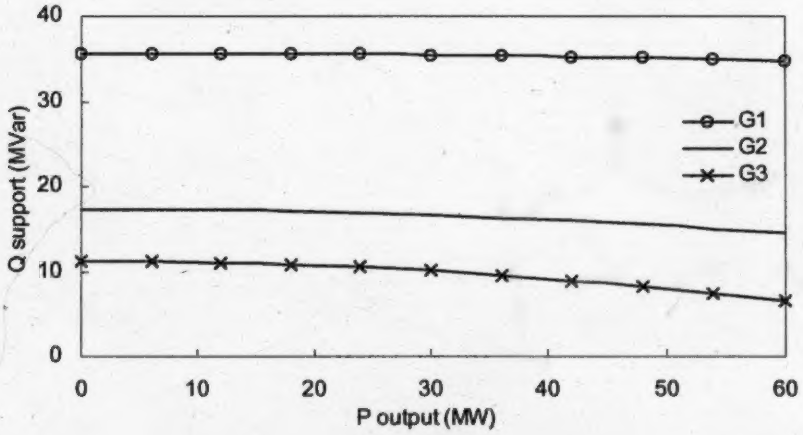


Figure 3.11: Comparison of Q support

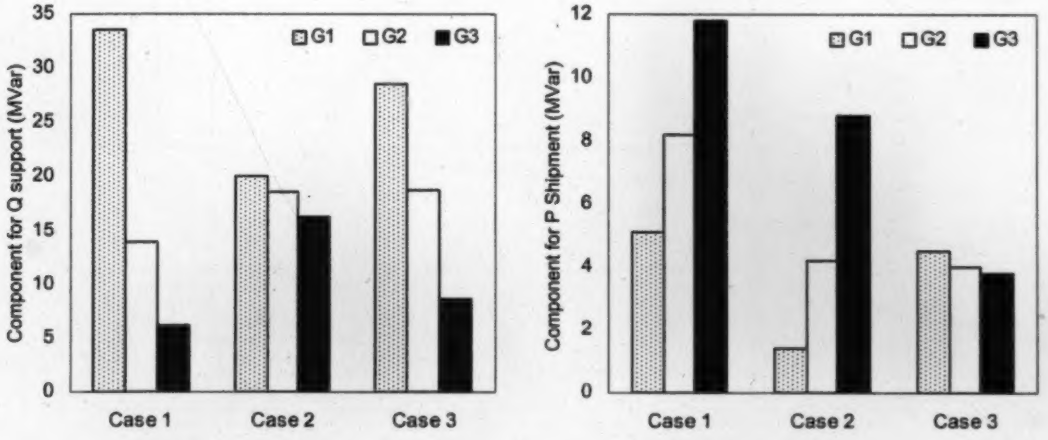
The above case study results can be verified using numerical studies. For this simple system, it is easy to separate the portion of reactive power support of the three generators that effectively reaches the power pool. Similar to Section 3.3, the total reactive output of each generator, and the portion of it arriving at the load bus can be monitored. The latter is just the $Q_{support}$ component, and the difference between this amount and the total reactive power output is the $Q_{P\text{-}shipment}$ component that is used for supporting the active power transmission of generators. In order to verify the results from the APR method and further illustrate the two components of reactive power output, three different cases are assessed. These cases are described as follows:

- Case 1: everything except the distance to the load center is the same. Generator 3 is the most distant generator, while Generator 1 is the closest.

- Case 2: everything except the active power output is the same. Generator 3 produces the highest active power, while Generator 1 provides the least.

- Case 3: everything except the terminal voltage is the same. Generator 1 has the highest voltage setting, while Generator 3 has the lowest.

Figure 3.12 plots the results of the separation for the three cases described here. For each case the load flow is solved, and the $Q_{P\text{-}shipment}$ and $Q_{support}$ components are obtained. The results in Figure 3.12 can be summarized as follows:



(a). Variation of $Q_{support}$ (b). Variation of $Q_{P\text{-}shipment}$

Figure 3.12: Variation of components $Q_{P\text{-}shipment}$ and $Q_{support}$

- Case 1: The closest generator (G1) needs less reactive power to transmit the same *P*, and it shows the largest $Q_{support}$ component. Generators G2 and G3 use most of their

Q output for transmitting their active power, and provide small amount of reactive power support to the system. Compared with Figure 3.11, the numerical results obtained here are consistent with those obtained from the APR method. The amounts of reactive power support of the three generators calculated from both methods are almost the same. This verifies the effectiveness of the method.

- Case 2: Generator G3 with the biggest active output also has the biggest Q output. However, most of it is used to transmit its P but not to support the system. The other generators with smaller P output have larger support components, and should receive more financial compensation for their reactive power support services.

- Case 3: G1, which has the highest voltage setting, provides more support than the other generators. However, by increasing its terminal voltage, G1 also increases its $Q_{P\text{-}shipment}$ component.

The studies performed on the five-bus system gives further demonstrations of the dual-functions of a generator's reactive power output. The features of generators and power systems that can significantly affect the support and shipment components are also assessed. These studies show clearly that the two components of a generator's reactive power output need to be separated from each other in order to realize the real fair competitive procurement of reactive power support.

3.5 Summary

Using some simple systems as examples, the characteristics of a generator's reactive power output are discussed in this chapter. The dual functions of a generator's reactive power output are analyzed and defined. An Active Power Reduction (APR) method is proposed to separate the dual functions. Study results are summarized as follows:

- Depending on the system parameters, the amount of active power and reactive power output of the generator, the generator can either support system security or draw reactive power support from the system.

- A generator's reactive power output has dual functions: one is to support system operation and security, the other is to transmit its own active power.

When the APR method is further tested in large power systems, the following problems have been found in this research: 1). Divergence problems can emerge in large systems. This is because the system may not be able to operate when the active power output of a major generator is reduced to zero. 2). The procedure of this method depends on load shedding. It is possible that load shedding could mask the component separation problem. These problems show that the APR method needs further development.

Chapter 4

Analysis of the Minimum Reactive Power Need of A Generator

Chapter 3 has presented one approach to analyze the dual function of a generator's reactive power output. This chapter investigates the problem from an alternative perspective. This perspective is to determine the minimum amount of reactive power required from a generator to support the transmission of its active power. Only when its reactive power output exceeds the minimum reactive power amount can the generator support system security. In this chapter, the concept of minimum reactive power need (Q_{min}) for generators is introduced. A technique is proposed to calculate the Q_{min} value.

4.1 Concept and Characteristics of Minimum Reactive Power Need

The minimum reactive power need — Q_{min} is defined as the least amount of reactive power needed from a generator to transmit its own active power without the help of the system. This concept is illustrated using the simplified system in Figure 4.1, where only one generator is supplying power to the system[7].

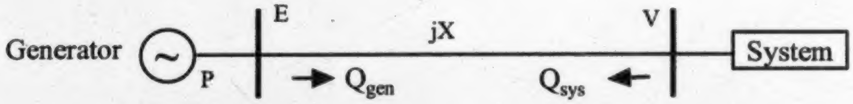


Figure 4.1: Diagram of single generator-system test system

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[7] Please refer to Footnote 1. Detailed data of this system are shown in Appendix B.

For a given active power output *P* and voltage setpoints *E* and *V*, the reactive power injections from the generator and the system are given by:

$$\begin{aligned} Q_{gen} &= \frac{E^2}{X} - \sqrt{\left(\frac{EV}{X}\right)^2 - P^2} \\ Q_{sys} &= \frac{V^2}{X} - \sqrt{\left(\frac{EV}{X}\right)^2 - P^2} \end{aligned} \tag{4.1}$$

The above equations can be rearranged as follows:

$$Q_{sys} = Q_{gen} + \frac{V^2}{X} - \frac{E^2}{X} \tag{4.2}$$

From Equation (4.1) and (4.2), it can be concluded that, depending on the voltage setting, both generator and system can inject reactive power into the transmission line. The reactive power injected is used to compensate the reactive losses related to the transmission of *P*. In order to transmit *P* without drawing reactive power support from the system, the following condition must be satisfied:

$$Q_{sys} \leq 0 \tag{4.3}$$

Using Equation (4.1), this condition can be expressed by:

$$\frac{V^2}{X} - \sqrt{\left(\frac{EV}{X}\right)^2 - P^2} \leq 0 \tag{4.4}$$

If the system side voltage *V* is assumed to be constant, Equation (4.4) can be rearranged as follows:

$$E \geq \sqrt{V^2 + \frac{P^2 X^2}{V^2}} \tag{4.5}$$

This equation presents the minimum voltage setting requirement. It shows that the generator has to set its terminal voltage higher than a certain value in order to transmit its own P without consuming any reactive power support from the system. Similarly, the corresponding minimum requirement for the generator's reactive power output can be obtained from Equation (4.2) and (4.5):

$$Q_{\min} = Q_{gen}\Big|_{Q_{sys}=0} = \frac{P^2}{V^2} X \tag{4.6}$$

Equation (4.6) represents the critical operation point where there is zero reactive power support consumed by the generator. If the generator outputs less reactive power than the Q_{min}, the transmission of P has to rely on the reactive power support from the system. The above analysis is further illustrated and verified using Figure 4.2. It is assumed that the generator in Figure 4.1 outputs 300MW, the line reactance is 0.1pu (base: 100MVA), and the system voltage V is set to 1.0 pu. The figure plots the generator and system reactive injections as generator terminal voltage E is increased.

Figure 4.2 shows that when generator voltage setting increases, generator reactive power output (Q_{gen}) increases, and the required reactive injection from the system (Q_{sys}) decreases. Two points in Figure 4.2 deserve special attention. The first one is Point A, which represents $E=V$. At this point, the reactive power injections from the system and the generator are the same: $Q_{sys}=Q_{gen}$, which means that each end of the transmission line supplies half of the reactive power losses. The second point of importance is Point B. At this point, where E is set to 1.044 pu, the required reactive power injection from the system side is equal to zero. It implies that for transmitting 300MW into the system, the generator should set its voltage to a minimum value of 1.044 pu in order not to 'tax' the system. The corresponding reactive power output of the generator, labeled as Q_{min}, is the minimum amount needed to support the generator's own active power transmission. If the generator outputs less reactive power than Q_{min}, it actually draws reactive power from the system and has a negative impact on system security; On the other hand, if the generator could produce reactive power greater than the Q_{min}, it provides reactive power support to the system. In Figure 4.2, the amount of reactive power support provided by the generator is marked as the shaded area.

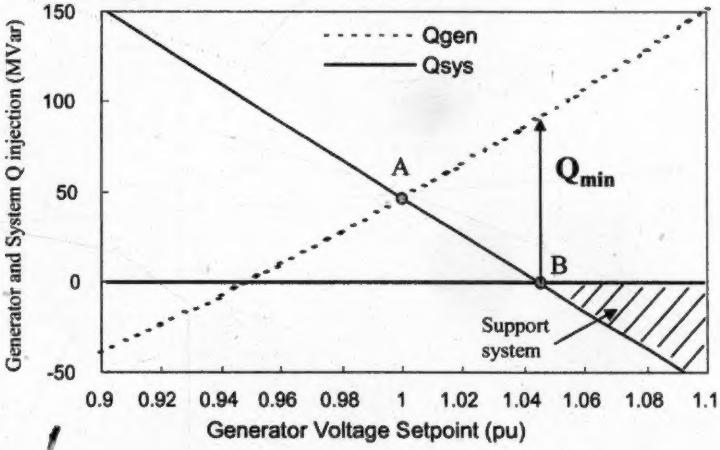
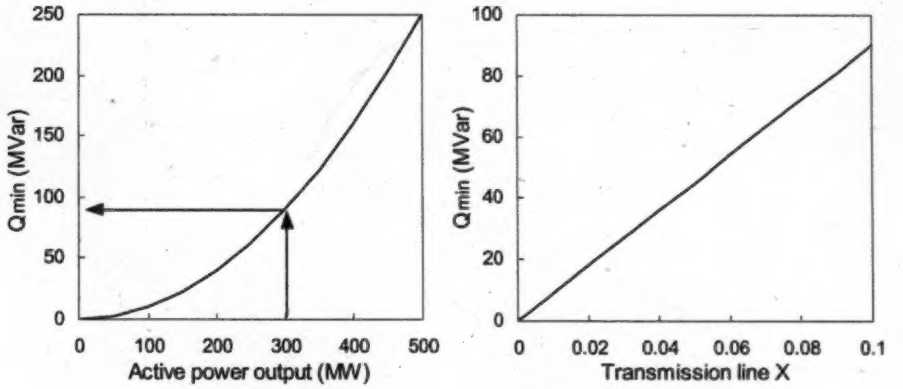


Figure 4.2: Distribution of Q injections from generator and system (Base: 100MVA)

Equation (4.6) shows that Q_{min} is a function of several factors such as the generator location and the amount of active power transmitted. Figure 4.3 (a) and (b) plot the characteristics of Q_{min} as a function of such factors. The figures clearly demonstrate that in order to transmit more active power, a larger Q_{min} is necessary. It is also shown that a distant generator requires a larger Q_{min} than the local generator to transmit the same amount of active power. Once the Q_{min} is available, the amount of system support service provided by a generator can be obtained by subtracting the Q_{min} from the total generator Q output.



(a). Function of active power (b). Function of reactance X(pu)

Figure 4.3: Characteristics of Q_{min}
(Base: 100MVA)

4.2 Method to Determine Q_{min}

The definition of Q_{min} in Section 4.1 is derived from a system with only one generator. In order to extent the concept to more complex power systems, the minimum reactive power need of a generator can be defined generally as the amount of reactive power produced by a given generator that has neither negative nor positive impact on system security level. The security level can be measured, for example, using the voltage stability margin (PV or QV curve margin). A method to determine the Q_{min} is proposed from this perspective.

4.2.1 The Concept of Reference Margin

In order to measure the negative or positive impact of a generator's reactive power output on system security, it is necessary to define a reference point for comparison first. The reference case can be defined as the case where the test generator (generator whose Q_{min} is to be found) doesn't enhance or reduce the system security level. The corresponding voltage stability margin is the reference margin. It is known that when the test generator has zero active and reactive power output, it has no impact on system security. One possible way to establish the reference case is, therefore, to set the active and reactive output of the test generator to zero, as shown in Figure 4.4.

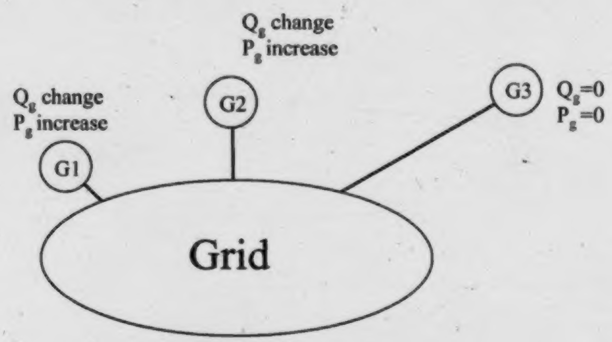


Figure 4.4: Illustration of the reference case

In Figure 4.4, Generator G3 is the test generator, which has zero output and therefore a neutral impact on system security. Since the system still needs to fulfill its load demand, the mismatched power is dispatched to other generators, such as G1 and G2. The voltage stability margin of this case is defined as the reference margin, shown as Curve A in Figure 4.5.

Curve A of Figure 4.5 represents the PV curve of the system when the test generator (G3) has zero active and reactive power output. If this generator starts to produce, say

100MW, active power without supplying any reactive power, the system must provide reactive power to support the generator. Consequently, the PV curve shrinks to curve C. On the other hand, if the generator can output 50MVar reactive power to the system, the system margin could increase to a level higher than the reference case (Curve B). This means that a portion of the 50MVar output is used to boost the system margin.

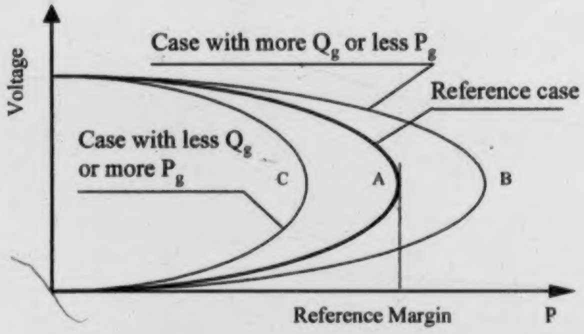


Figure 4.5: Illustration of the reference margin

The system margin therefore varies with the amount of reactive power produced by the generator. Figure 4.6 shows the situation more clearly. In this figure, the curve represents the system margin as affected by the reactive output of the test generator. The curve has a positive slope since increased reactive output results in increased margin. At point A, the system margin is just equal to the reference margin. It means that the generator has a neutral impact on the system at this point. The corresponding reactive power output of the generator is, therefore, Q_{min} for the given 100MW active power transmission level. If the generator outputs less reactive power than the Q_{min}, such as Point C in Figure 4.6, it has a negative impact on system security since the margin is less than the reference margin. On the other hand, if the generator produces more reactive power than the Q_{min}, such as Point B, it contributes to improving system security and should be compensated for the reactive power support provided.

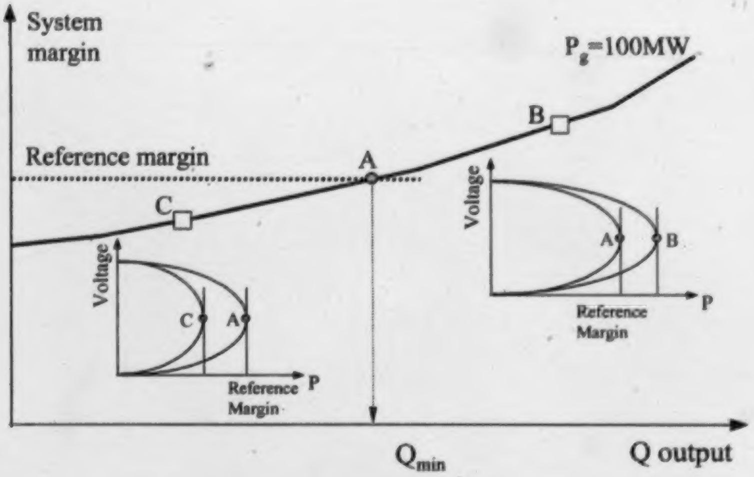


Figure 4.6: *Margin* ~ Q_{output} curve

4.2.2 Determination of Q_{min} versus Generator Active Power Output Curve

When the test generator produces different P output, the *Margin* ~ Q_{output} curve of Figure 4.6 will be a family of curves, each representing a different P output level, as shown in Figure 4.7.

The intersection of the reference margin line with each *Margin* ~ Q_{output} curve indicates the point where the generator's reactive output has yielded the same amount of margin as the reference margin for the given P output level. The corresponding reactive output is the Q_{min} value for the specific active power output level. Therefore, the Q_{min} versus generator active power output curve in the form of Figure 4.8 can be obtained.

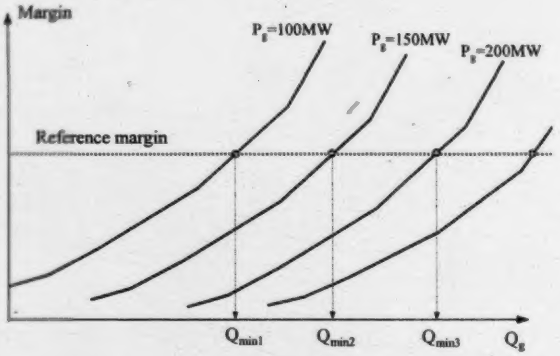


Figure 4.7: Family of *Margin* ~ Q_{output} curves

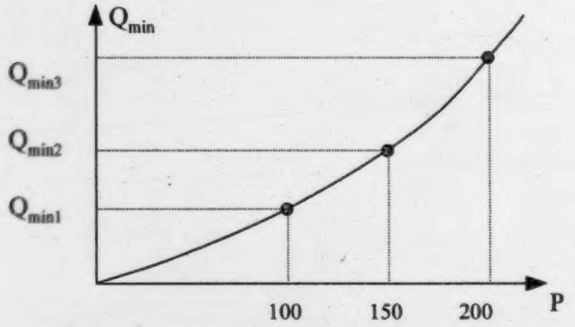


Figure 4.8: $Q_{min} \sim P_{output}$ curve

4.2.3 Mathematical Description

The above procedure of determining Q_{min} for a particular generator (the test generator) can also be mathematically described as follows. The impact of the test generator's active and reactive power output can be symbolically described as:

$$M_i = f(P_i, Q_i) \tag{4.7}$$

where M_i is an index to measure the impact of the i^{th} generator on the system. Here the voltage stability margin is used as the index to measure the impact. P_i and Q_i represent the active and reactive power output of the test generator, respectively.

If the test generator does not output any active or reactive power, the generator can be considered as having a zero impact on the system, the system margin would be

$$M_{i0} = f(P_i = 0, \quad Q_i = 0) \tag{4.8}$$

This margin M_{i0} is called the reference margin since it can be used to assess the degree of margin variations. If the test generator outputs some active power, P_i, the margin will change. The reactive power output of the generator can be adjusted to 'neutralize' the impact as follows:

$$M_i = f(P_i, Q_{ix}) = M_{i0} \tag{4.9}$$

where Q_{ix} is the amount of reactive power needed from the generator to balance the impact of P_i on the margin (i.e. maintaining a system margin of M_{i0}). Q_{ix} can be solved from the above equation and be expressed as

$$Q_{ix} = g(P_i, M_{i0}) \tag{4.10}$$

Here Q_{ix} is in effect the minimum reactive power output required for the test generator, given its active power output level of P_i. If the generator actually outputs $P_{i\text{-}output}$ for a particular study case, the Q_{min} of this generator for this case would be

$$Q_{i-\min} = g(P_i = P_{i-output}, M_{i0}) \tag{4.11}$$

4.2.4 An Analytical Example

To illustrate the proposed method, the three-bus system shown in Figure 3.3 is used as an example here. The procedure for Q_{min} determination is as follows:

1). Establish the reference case

Generator G1 is the test generator in this case. The reference case is established by setting the active and reactive power output of G1 to zero, i.e. $P_1=Q_1=0$. The active power output of G2 is increased so that the supply-demand remains balanced, i.e. $P_2=P_L$. the reference case thus becomes the system shown in Figure 4.9.

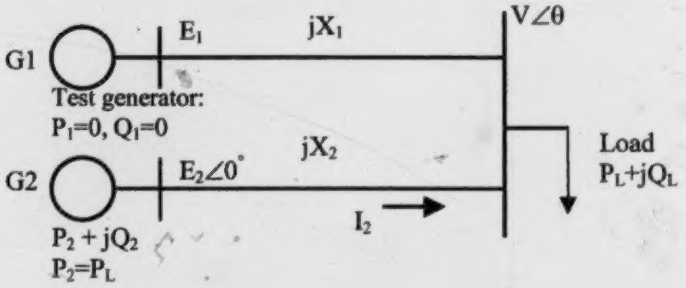


Figure 4.9: Three-bus system reference case

2). Calculate the reference margin

The reference margin is given by the nose point of the PV curve, which represents the transfer capability of the system. The PV curve of the reference case is obtained by stressing the Q load only. It is derived as follows:

$$\begin{aligned}\overline{S_L} &= P_L + jQ_L = \overline{V}\,\overline{I_2}^{*} \\ &= \overline{V}\left(\frac{\overline{E_2} - \overline{V}}{jX_2}\right)^{*} = \frac{1}{-jX_2}\left(\overline{V}\,\overline{E_2}^{*} - V^2\right) \\ &= \frac{j}{X_2}\left[E_2 V(\cos\theta + j\sin\theta) - V^2\right] \\ &= -\frac{E_2 V}{X_2}\sin\theta + j\left(\frac{E_2 V}{X_2}\cos\theta - \frac{V^2}{X_2}\right)\end{aligned} \tag{4.12}$$

Therefore,

$$\begin{aligned} P_L &= -\frac{E_2 V}{X_2}\sin\theta \\ Q_L &= \frac{E_2 V}{X_2}\cos\theta - \frac{V^2}{X_2}\end{aligned} \tag{4.13}$$

Combine the two equations together and eliminate load bus voltage angle θ, it yields

$$P_L + \left(Q_L + \frac{V^2}{X_2}\right)^2 = \left(\frac{E_2 V}{X_2}\right)^2 \tag{4.14}$$

Rearrange the above equation, the following equation with voltage V as the variable is obtained,

$$V^4 + (2Q_L X_2 - E_2^2)V^2 + X_2^2(P_L^2 + Q_L^2) = 0 \tag{4.15}$$

From the knowledge of voltage stability, the maximum transfer occurs when the two solutions of voltage merge into one. That is,

$$\begin{aligned}&\Delta = 0 \Rightarrow \\ &(2Q_L X_2 - E_2^2)^2 - 4X_2^2(P_L^2 + Q_L^2) = 0\end{aligned} \tag{4.16}$$

Thus, the reference margin is

$$Q_{m.ref} = \frac{E_2^2 - 4X_2^2 P_L^2}{4X_2 E_2^2} \tag{4.17}$$

3). Determine Q_{min}

In order to calculate Q_{min} for the test case, as shown in Figure 4.10, the active power output of the test generator needs to be frozen and the reactive power needs to be changed. When the reactive power output is changed, the margin of the system will change correspondingly. Q_{min} can be obtained when the voltage stability margin of the test case is equal to the reference margin.

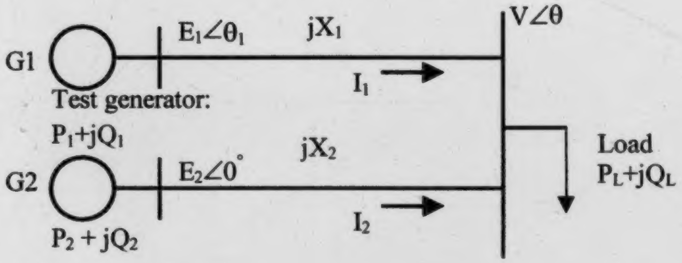


Figure 4.10: Three-bus system test case

The voltage stability margin of the test case is derived as follows. For generator G1 and G2, the following load flow equations exist:

$$\begin{aligned}
\overline{S_2} &= P_2 + jQ_2 = \overline{E_2}\,\overline{I_2}^{*} \\
&= \overline{E_2}\left(\frac{\overline{E_2} - \overline{V}}{jX_2}\right)^{*} = \frac{1}{-jX_2}\left(E_2^2 - \overline{E_2}\,\overline{V}^{*}\right) \\
&= \frac{j}{X_2}\left[E_2^2 - E_2 V(\cos\theta - j\sin\theta)\right] \\
&= -\frac{E_2 V}{X_2}\sin\theta + j\left(\frac{E_2^2}{X_2} - \frac{E_2 V}{X_2}\cos\theta\right)
\end{aligned} \tag{4.18}$$

$$\begin{aligned}
\overline{S_1} &= P_1 + jQ_1 = \overline{E_1}\,\overline{I_1}^{*} \\
&= \overline{E_1}\left(\frac{\overline{E_1}-\overline{V}}{jX_1}\right)^{*} = \frac{1}{-jX_1}\left(E_1^2 - \overline{E_1}\,\overline{V}^{*}\right) \\
&= \frac{j}{X_1}[E_1^2 - E_1V(\cos\theta_1 + j\sin\theta_1)*(\cos\theta - j\sin\theta)] \\
&= -\frac{E_1V}{X_1}(\sin\theta_1\cos\theta - \cos\theta_1\sin\theta) \\
&\quad + j[\frac{E_1^2}{X_1} - \frac{E_1V}{X_1}(\cos\theta_1\cos\theta + \sin\theta_1\sin\theta)]
\end{aligned} \tag{4.19}$$

Therefore,

$$\begin{aligned}
P_1 &= -\frac{E_1V}{X_1}(\sin\theta_1\cos\theta - \cos\theta_1\sin\theta) \\
Q_1 &= \frac{E_1^2}{X_1} - \frac{E_1V}{X_1}(\cos\theta_1\cos\theta + \sin\theta_1\sin\theta) \\
P_2 &= -\frac{E_2V}{X_2}\sin\theta \\
Q_2 &= \frac{E_2^2}{X_2} - \frac{E_2V}{X_2}\cos\theta
\end{aligned} \tag{4.20}$$

Considering $P_L=P_1+P_2$, there exist,

$$\begin{aligned}
P_L &= P_1 - \frac{E_2V}{X_2}\sin\theta \\
Q_L &= \frac{E_1^2}{X_1} - Q_1 - \frac{V^2}{X_1} + \frac{1}{X_2}(E_2V\cos\theta - V^2)
\end{aligned} \tag{4.21}$$

Combine the two equations together and eliminate θ, again an equation for variable V is obtained:

$$\begin{aligned} & V^4\left(\frac{1}{X_1}+\frac{1}{X_2}\right)^2+\left[2\left(Q_L+Q_1-\frac{E_1^2}{X_1}\right)\left(\frac{1}{X_1}+\frac{1}{X_2}\right)-\frac{E_2^2}{X_2}\right]V^2+ \\ & \quad (P_1-P_L)^2+\left(Q_L+Q_1-\frac{E_1^2}{X_1}\right)^2=0 \end{aligned} \tag{4.22}$$

Similarly, the maximum transfer occurs when the two solutions of voltage merge into one. That is,

$$\begin{aligned} & \Delta=0 \Rightarrow \\ & \frac{E_2^4}{X_2^4}-4\left(Q_L+Q_1-\frac{E_1^2}{X_1}\right)\left(\frac{1}{X_1}+\frac{1}{X_2}\right)\frac{E_2^2}{X_2^2}-4(P_1-P_L)^2\left(\frac{1}{X_1}+\frac{1}{X_2}\right)^2=0 \end{aligned} \tag{4.23}$$

Re-arrange Equation (4.23), the margin of the test case is

$$Q_{m.test}=\frac{E_2^2X_1}{4X_2(X_1+X_2)}-\frac{X_2(X_1+X_2)}{E_2^2X_1}(P_1-P_L)^2+\frac{E_1^2}{X_1}-Q_1 \tag{4.24}$$

When $Q_{m.test}$ is equal to $Q_{m.ref}$, the test case has the same margin as the reference margin. As a result, Q_1 under such condition is the required Q_{min}. Thus, Q_{min} is obtained as in the following equation:

$$\begin{aligned} & Q_{m.ref}=Q_{m.test} \Rightarrow \\ & Q_{\min}=\frac{E_2^2X_1}{4X_2(X_1+X_2)}-\frac{X_2(X_1+X_2)}{E_2^2X_1}(P_1-P_L)^2+\frac{E_1^2}{X_1}-\frac{E_2^2-4X_2^2P_L^2}{4X_2E_2^2} \end{aligned} \tag{4.25}$$

4). Numerical results

Equation (4.25) shows that the Q_{min} is a function of the voltage setting E, the electrical distance X and the active power P. If there are changes in any of these parameters, a different Q_{min} is expected. This phenomenon is tested numerically, using the three-bus system. The parameters are as follows: E_1=E_2=1.0pu, X_1=0.1pu, X_2=0.2pu, P_L=1.0pu, Q_L=0.5pu and P_1=0.5pu. P_1 and X_1 are varied respectively and the Q_{min} values are obtained according to Equation (4.25). Results are shown in Figure 4.11.

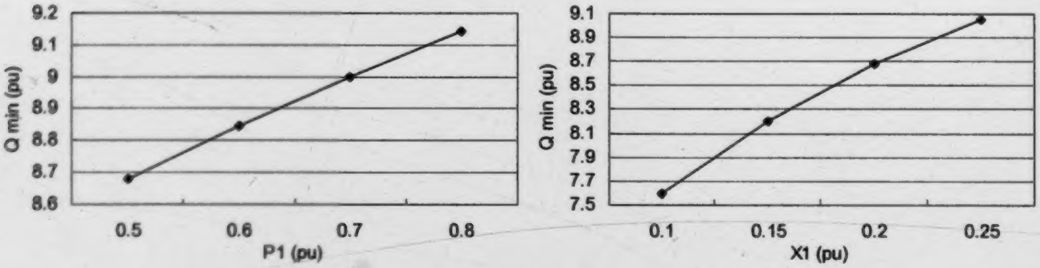


Figure 4.11: Q_{min} of the three-bus test sytem

(Base: 100MVA)

Figure 4.11 shows the relationship between Q_{min} and active power P, electric distance X, respectively. It shows that a higher Q_{min} is required if the generator wants to transmit more active power P to the system. Similarly, a distant generator also needs to have a high Q_{min} to transmit its own active power. The further the generator is, the larger the Q_{min} is needed. These test results are consistent with the intuition and analysis.

4.3 Numerical Method to Determine Q_{min}

It is very difficult, if not impossible, to derive an analytical method to solve Q_{min} for actual power systems because the systems are complex and the location of load centers cannot be easily determined. Here a numerical version of the method presented in Section 4.2 is proposed. The problem of finding Q_{min} can be cast into two sub-problems. The first one is to establish a reference margin and the second one is to establish the *Margin* ~ Q_{output} curve with a given active power output level. The calculation of Q_{min} can be carried out according to the following procedure:

1). Establish the base case by which a generator's Q_{min} needs to be determined.

2). Select the generator whose Q_{min} is to be determined. This generator is called the test generator.

3). Create the reference case by a) setting the active and reactive power output of the test generator to zero and b) re-dispatching the active power output of the remaining generators so that the supply-demand remains balanced.

4). The PV curve of the reference case is then obtained without the participation of the test generator. The nose point of the PV curve provides the reference margin.

5). Create a set of cases where all generators remain the same active power output levels and the reactive power output of the test generator is varied in steps. Each step represents one case.

6). For each case, compute its PV curve and the associated margin. The results, margin as a function of the Q output level of the test generator, form one data point on the *Margin* ~ Q_{output} curve.

7). Based on the *Margin* ~ Q_{output} curve and using the reference margin value, Q_{min} is determined for the given generator output level.

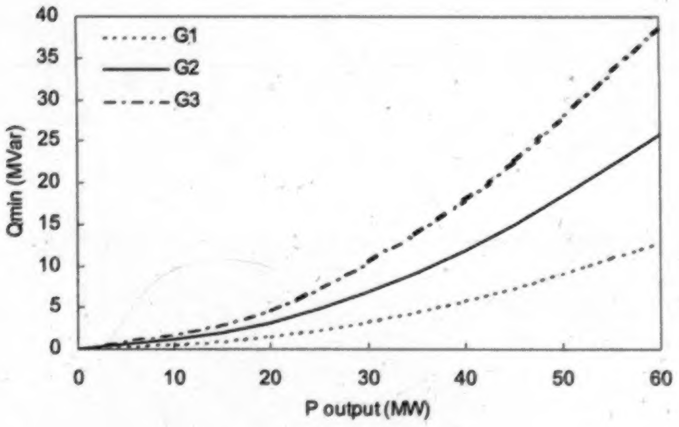
8). If the Q_{min} values for other generators need to be determined, steps 2 to 7 can be repeated. If the Q_{min} value needs to determined for the same generator but with a different active power output level, steps 1 to 7 needs to be repeated.

It is possible to develop other numerical methods to solve for Equation (4.25). The above procedure is the one adopted in this thesis. This procedure is applied to compute Q_{min} values for the five-bus system shown in Figure 3.7. Two different scenarios are assessed in this study:

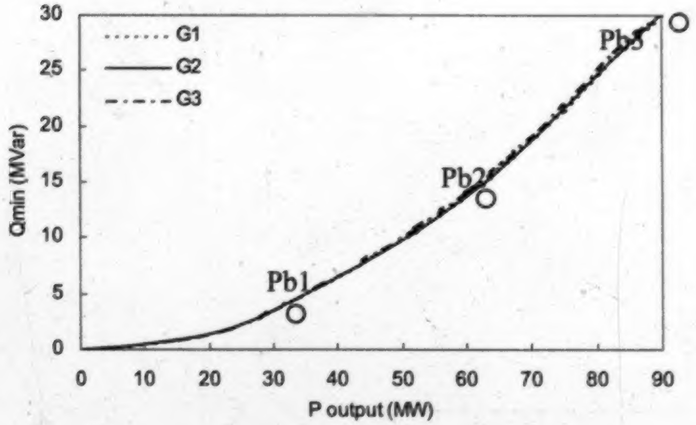
- Case 1: everything except the distance to the load center is the same. Generator 3 is the most distant generator, while Generator 1 is the closest.
- Case 2: everything except the active power output is the same. Generator 3 produces the highest active power, while Generator 1 provides the least.

Test results for the above two cases are shown in Figure 4.12 (a) and (b), respectively. The result of Case 1 shows that the closest generator G1 needs the least Q_{min} to transmit the same amount of P, while the most distant generator G3 needs the highest Q_{min}. This result is consistent with intuition. Since more reactive power is consumed on longer transmission lines, it is reasonable to expect higher Q_{min} for the distant generator.

Test result of Case 2 shows that the $Q_{min} \sim P_{output}$ curves of the three generators are almost identical. This is true since there is no difference between the three generators except the amount of base case active power output (*Pb1*, *Pb2* and *Pb3*). Generator 3 has the largest active output in the base case and consequently has the biggest Q_{min} among the three generators. This result also agrees well with common understanding. It further verifies the relationship between Q_{min} and active power output: when more active power needs to be transmitted, a higher Q_{min} is expected.



(a). Case 1: *X3>X2>X1*

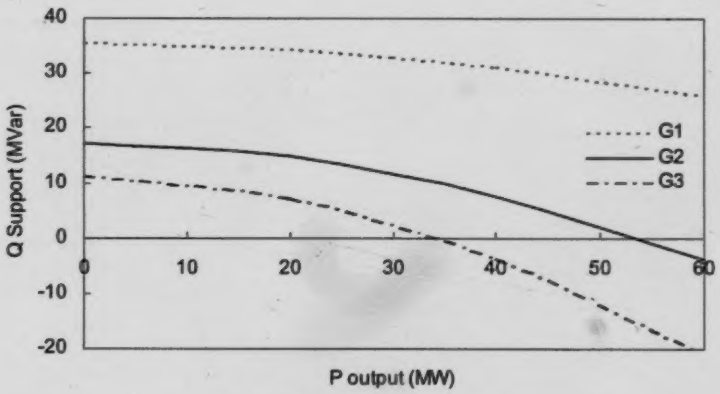


(b). Case 2: *Pb3>Pb2>Pb1*

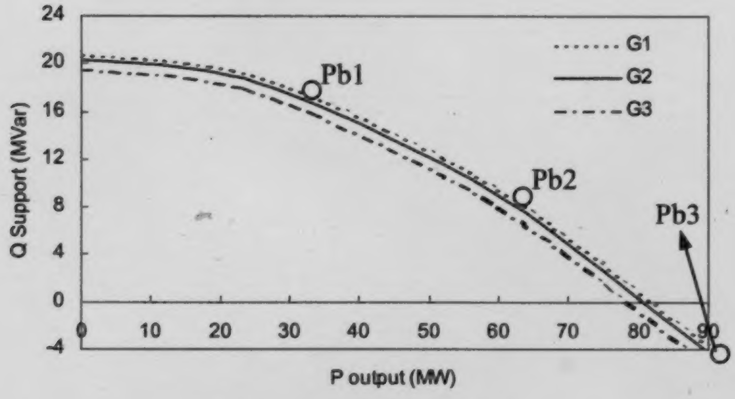
Figure 4.12: $Q_{min} \sim P_{output}$ curves of the three generators

4.4 Comparison with the Active Power Reduction Method

Two methods, the Active Power Reduction (APR) method and the Q_{min} method, are presented in Chapter 3 and 4, respectively. Both methods have their own characteristics and should be compared. The comparison is carried out using the five-bus system shown in Figure 3.7 as a study case. The reactive power support levels of the three generators are first calculated using the Q_{min} method for both Case 1 and Case 2. The results are shown in the following figures:



(a). Case 1: *X3>X2>X1*



(b). Case 2: *Pb3>Pb2>Pb1*

Figure 4.13: Reactive power support of the three generators using the Q_{min} method

Figure 4.13(a) shows that the closest generator G1 provides the largest reactive power support to the system. The most distant generator G3 provides the least support. When G3 needs to transmit more active power than 35MW, its reactive power support to the system is negative, i.e. it has to draw reactive power from the system instead. Figure

4.13(b) shows that the three generators are almost equally effective in providing reactive power support when they have the same electrical distance from the load center. The above results are then compared with those obtained from the APR method, as shown in Figure 4.14 and 4.15. The comparison results are summarized as follows:

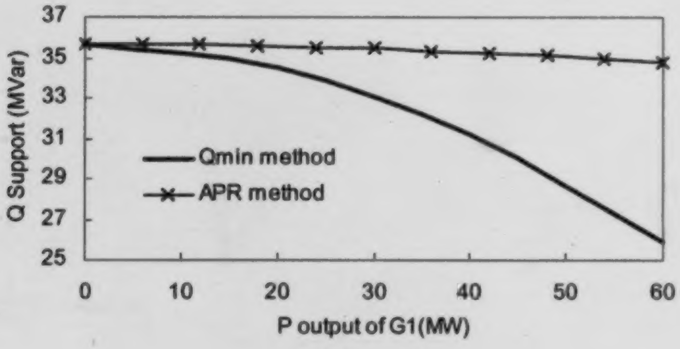
- The two methods show an overall consistency. Results of the APR method show that the local generator G1 provides the highest reactive power support while G3, the most distant one, provides the least. This is consistent with the test results obtained from the Q_{min} method (shown in Figure 4.13(a)). Similarly, results obtained from both methods consistently indicate that when the three generators have the same electrical distance to the load center, the amounts of reactive power support they provide are different only if they output different amount of active power.

- There exist numerical differences between the results obtained from the two methods. Reactive power support results obtained from the Q_{min} method are usually smaller than those from the APR method. The difference is due to the different formulations of the problem. The Q_{min} method uses power transfer capability to measure the reactive power support level while the APR method is concerned with the variation of a generator's reactive power output as affected by the test generator.

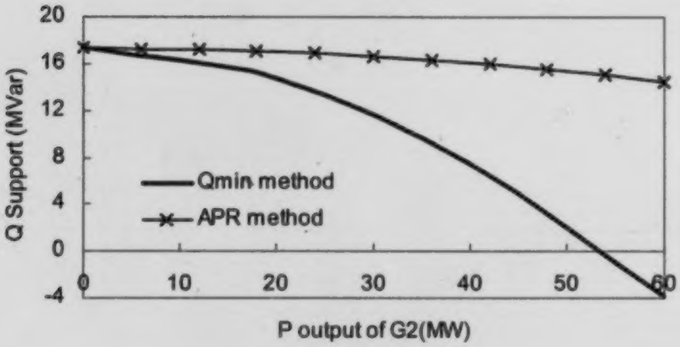
- The APR method is more straightforward compared with the Q_{min} method. Once the method is successfully carried out, it gives a direct separation of the $Q_{support}$ component and the $Q_{shipment}$ component. The Q_{min} method, on the other hand, needs at least one more step to give an explicit indication of the amount of reactive power support being provided.

- The Q_{min} method has the advantage of using a standard index to measure the impact of reactive power. Unlike the APR method and many other published methods, the Q_{min} method tries to assess the value of a generator's reactive power output from the perspective of its impact on the system capability level, which is the main purpose to procure reactive support service.

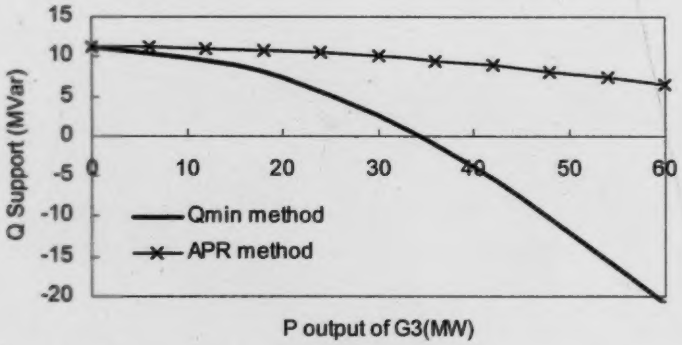
- The Q_{min} method is superior to the APR method in terms of numerical performance. Test results show that divergence problems can be found depending on the adopted procedure for load shedding in the APR method. The divergence problem, however, is not present when the Q_{min} method is used. Furthermore, the need for load shedding can be avoided in the Q_{min} method. Therefore, the Q_{min} method is more suitable for large-scale power systems.



(a). Comparison of G1



(b). Comparison of G2



(c). Comparison of G3

Figure 4.14: Q_{min} method compared with APR method for case *X3>X2>X1*

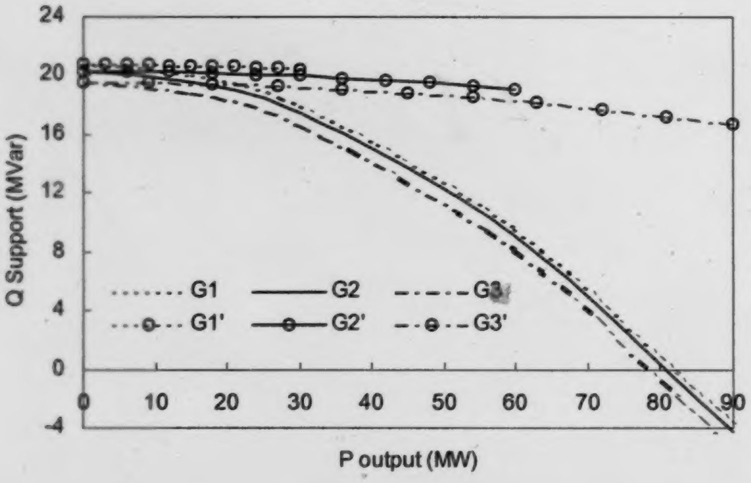


Figure 4.15: Q_{min} method compared with APR method for case *Pb3>Pb2>Pb1*

(G1,G2,G3 are obtained using the Q_{min} method,

G1′,G2′,G3′ are obtained using the APR method)

4.5 Summary

The concept of Q_{min} is introduced to assess the reactive power support service from generators. Q_{min} is defined as the least amount of reactive power needed from each generator to transmit its own active power without the help of the system. A practical method to determine the Q_{min} is developed. This method utilizes voltage stability analysis tools. Based on case studies using the proposed method and comparisons with the APR method, the following conclusions are reached:

- Q_{min} represents the amount of reactive power helping the transmission of a generator's active power output. Reactive power support from a generator is available when its output exceeds Q_{min}. Otherwise, the system supports the generator instead.

- Q_{min} is a function of several factors, such as active power P, line reactance X etc. A higher Q_{min} is required when more active power is transmitted or the generator is far away from the load center.

- Compared to the APR method, the Q_{min} method is more suitable for large-scale power systems. Furthermore, the Q_{min} method assesses the value of reactive power support using a voltage stability approach, which provides more complete and useful information for measuring a generator's reactive power support services.

Chapter 5

Generalized Q_{min} Method and Case Study Results

Chapter 4 proposes the Q_{min} method for calculating the reactive power support service needs of generators. The method is tested on actual large systems in this chapter. Some practical implementation issues are addressed first. A generalized Q_{min} method is then developed based on these considerations. The method is tested on both the BC Hydro and the Alberta systems. Sensitivity studies are performed to test the robustness of the method.

5.1 Implementation Issues of the Q_{min} Method

When implementing the proposed Q_{min} method of Chapter 4 in actual large systems, there exist several practical issues that must be addressed. These practical considerations are discussed in this section.

5.1.1 Tools to Measure System Margin

The proposed method uses the voltage stability margin to quantify the impact of a generator on the system. Although other indices could also be used, years of industry experience have shown that the voltage stability margin is the most effective tool to measure the capability of a power transmission system. There are several approaches to obtain the voltage stability margin. The PV curve and QV curve approaches are the two most commonly used methods. The effectiveness of the two methods in the Q_{min} method is illustrated as follows.

1). PV curve method

This method is introduced in Chapter 2. According to the procedure of determining the Q_{min} value of a given generator, changes in the load pattern and generator active power output should be avoided. This requirement reveals a disadvantage of using the PV curve approach in the Q_{min} method, since changes in active power load and generation are always necessary when using the PV curve method. This disadvantage is shown in Figure 5.1, where the five-bus system in Figure 3.7 is used and Generator 3 is selected as the test generator. The following system parameters are based on 100MVA: P_1=60MW, P_2=120MW, P_3=180MW, P_L=360MW, Q_L=160MVar, X_1=0.1pu, X_2=0.2pu, X_3=0.3pu.

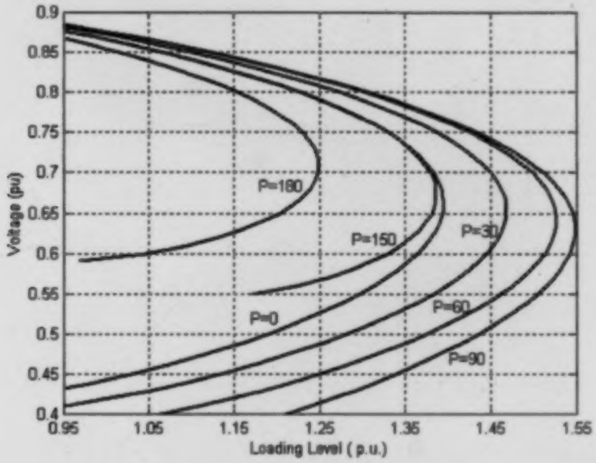


Figure 5.1: Impact of active power output on PV curve margin

The curves in Figure 5.1 are all PV curves obtained when setting the active power output of Generator 3 to different values. The maximum loading level (also called nose point of the PV curve) of each curve represents the maximum power transfer capability of the system compared with the base case load level. Figure 5.1 shows, that when the active power output of Generator 3 is increased to 30MW, 60MW and 90MW, respectively, the system power transfer capability increases. However, further increase in the active power output causes the power transfer capability to decrease, shown by curves *P*=150MW and *P*=180MW.

The decrease of system margin can be explained using Figure 5.2, where the reactive power output of the three generators is monitored. Curve P_{max} of Figure 5.2 is composed of all the nose points in Figure 5.1, representing the maximum power transfer capabilities for different active power output levels of Generator 3. Curves Q1, Q2 and Q3 record the reactive power output of the three generators at the nose points. The curve 'Total *Q*' is the sum of these three curves. Figure 5.2 shows that the P_{max} first increases and then decreases, which is consistent with the changes in total *Q* output. As a result, it can be concluded for this phenomenon that the system margin increases when more reactive power is produced or vice versa.

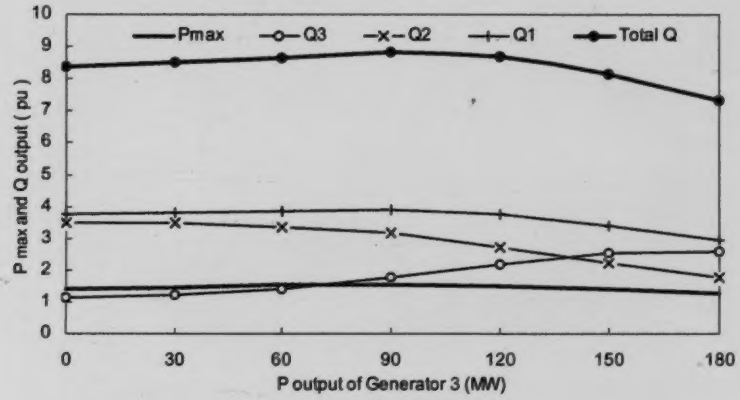


Figure 5.2: *Q* output and power transfer capability

This example shows that it is not effective to use a PV curve in the Q_{min} method as a tool to measure the voltage stability margin. This is because it is inevitable to change active power load and generation in forming the PV curve. The changes of active power then lead to great changes in load pattern and reactive power output of the generators, which are not desirable in Q_{min} calculation.

2). QV curve method

The traditional procedure of obtaining a QV curve is presented as follows [34]: 1). Add a fictitious synchronous condenser (SC) to the test load bus; 2). Change the voltage setting of the condenser; 3). Monitor the reactive power output of the fictitious synchronous condenser with respect to the voltage setting and thus form the QV curve. A family of QV curves can be obtained when a generator's active power output is changed to different values. Figure 5.3 shows the typical QV curves of a system when a generator produces at different active power levels. Point A represents the nose point when the generator has zero active power output. When the generator produces P_1, the nose point shrinks to B. Thus ΔM is the corresponding change in the QV curve margin.

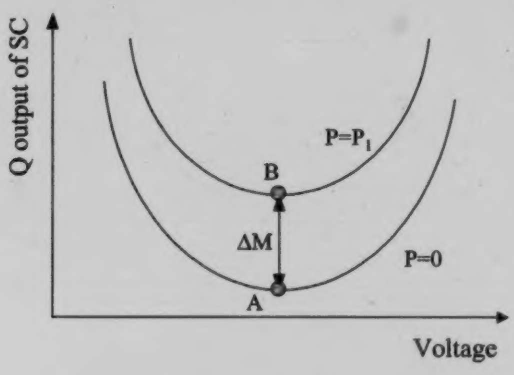


Figure 5.3: Sample QV curve

Similarly, the QV curves are obtained for the five-bus system in Figure 3.7. The active power output levels of the three generators are changed respectively, and the corresponding QV curve margins are recorded, as shown in Figure 5.4. Each point of the curves in Figure 5.4 represents one nose point of a QV curve. Therefore, these curves illustrate the changes of system power transfer capability with respect to different active power output of the generators. Figure 5.4 shows that in this sample system the power transfer capability decreases when more active power is provided.

The QV curve method has the advantage of avoiding active power changes in the scaling process. Thus it has less impact on active power load pattern. The limitation of this method resides in the arbitrariness in the selection of the test load bus. There is the possibility of missing key information.

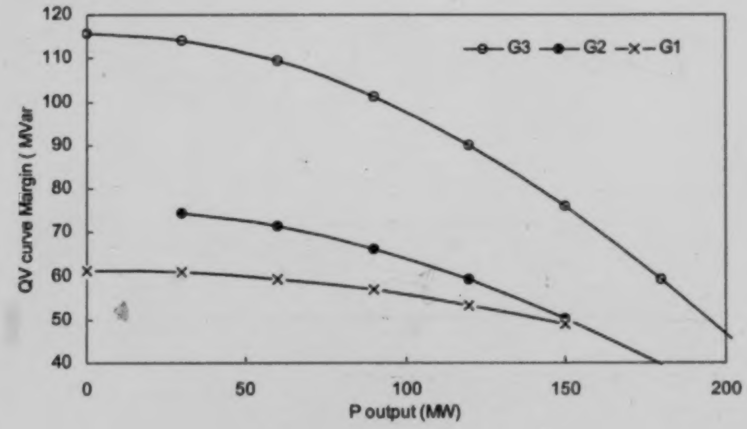


Figure 5.4: QV curve margin w.r.t. *P* output

The QV curve method can be extended to multiple buses. In this variation, the reactive power loads at selected buses are increased to a level when the system experiences voltage collapse. This method is called the Q-stress method, implying that the *Q* load is used to stress (test) the system. The PV curve method scales up both the *P* and *Q* loads and is called the S-stress method in some literature. The Q-stress method is needed for Q_{min} determination since it maintains the same active power generation pattern for all generators. Various voltage stability research works have shown that either the Q-stress or S-stress methods can be used to measure system margin, as long as they are used consistently [34].

5.1.2 Generation Re-dispatching Scheme.

Step 3 of the Q_{min} calculation procedure is to form the reference case by setting the active and reactive power of the test generator to zero. This change produces power mismatch. The remaining generators need to be re-dispatched to balance the load-generation mismatch. The following are possible generation re-dispatching schemes:

- Scheme 1: Proportional dispatch: This scheme increases the output of the remaining generators in proportion to the base case active power output levels of the generators.
- Scheme 2: Optimal power flow dispatch: This scheme uses OPF as a tool to determine how the mismatched power is dispatched among the remaining generators. The justification is that after the test generator is put off line, the operator would adjust the system in such a way to achieve optimal performance.
- Scheme 3: Dispatch according to an operating guide: This scheme mimics the actual system response when a generator is left out of service. Most companies rely on a set of operating guides to establish new operating conditions. The guide could be developed based on OPF results. In this case, Scheme 3 is the same as Scheme 2.

Both Schemes 1 and 2 have been tested in this thesis. As long as the same scheme is used for all generators, the results will be consistent and comparable. The scheme of load shedding was also tested. This scheme sheds a certain amount of load according to the size of the test generator in order to reach a new power balance. It was found that this scheme is not justified from both theoretical and practical perspectives. In practice if a generator were unsuccessful to bid to operate, it would be expected that other generators were to pick up the power mismatch.

5.1.3 Problem of Large Power Plants.

A power plant is usually composed of several generators. In theory, the power output of the whole plant should be reduced to zero when establishing the reference case. This may not be possible for some large plants. To solve this problem, the total output of such a plant is scaled down in steps. The Q_{min} value is then obtained through extrapolation of the results.

5.2 Flow Chart of the Generalized Q_{min} Method

The basic method is improved by taking into account the practical implementation issues. The resulting generalized method, suitable for actual power systems, to determine Q_{min} is shown in Figure 5.5. Based on this flow chart, the generalized method was then programmed using IPLAN, a programming language of Power Technologies (PTI) and was implemented under PTI Power System Simulator for Engineering (PSS/E) environment to calculate the Q_{min} values.

Although radial networks are used in previous chapters to illustrate a generator's dual functions and reactive power support need, and to introduce the concept of Q_{min}, the generalized Q_{min} method is independent of network topology. In other words, the generalized Q_{min} method is not limited to radial networks, but suitable for large actual power systems. Since this method is based on a commonly used voltage stability assessment tool, the Q-stress approach, all system parameters such as line resistance and susceptance are modeled in the method. The Q_{min} results calculated in this chapter and the subsequent chapters include the impact of such parameters.

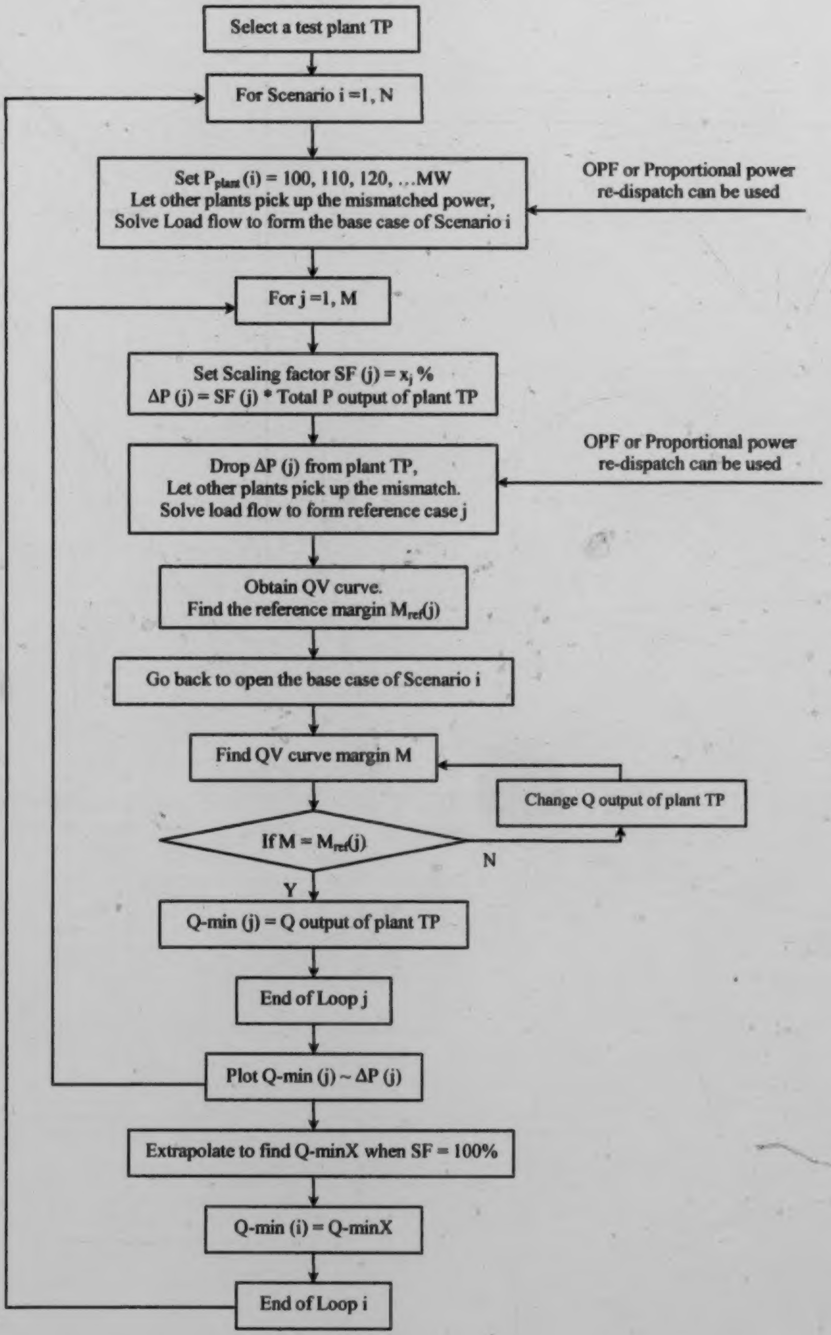

Select a test plant TP
For Scenario i =1, N
Set P_{plant} (i) = 100, 110, 120, …MW
Let other plants pick up the mismatched power,
Solve Load flow to form the base case of Scenario i
OPF or Proportional power re-dispatch can be used
For j =1, M
Set Scaling factor SF (j) = x_j %
ΔP (j) = SF (j) * Total P output of plant TP
Drop ΔP (j) from plant TP,
Let other plants pick up the mismatch.
Solve load flow to form reference case j
OPF or Proportional power re-dispatch can be used
Obtain QV curve.
Find the reference margin M_{ref}(j)
Go back to open the base case of Scenario i
Find QV curve margin M
Change Q output of plant TP
If M = M_{ref}(j)
N
Y
Q-min (j) = Q output of plant TP
End of Loop j
Plot Q-min (j) ~ ΔP (j)
Extrapolate to find Q-minX when SF = 100%
Q-min (i) = Q-minX
End of Loop i

Figure 5.5: Flow chart of the generalized Q_{min} method

5.3 BC Hydro Test System

The proposed Q_{min} method is tested on two actual large power systems. The first system is the BC Hydro system. The BC Hydro system, obtained from BC Hydro, has 1044 buses, 108 power plants and 1426 branches. The total generation is 9242MW and the total load is 8689 MW. Large plants such as GMS, REV, MCA and KCL supply power through long distance 500kV lines to the load center ING. Meanwhile, small generators such as BUT and BR2 also transmit power through a much shorter distance to the load center. Figure 5.6 shows the reduced diagram of the full BC Hydro system and its key plants. Geographic structure of the BC Hydro system is shown in Appendix A. Table 5.1 lists the capacities of major plants of the BC Hydro system.

Table 5.1: Capicities of major power plants of the BC Hydro system

Plant Name	**GMS**	**REV**	**MCA**	**PCN**	**SEV**	**KCL**	**BR2**	**BR1**
No. of generators	10	4	4	4	3	4	4	4
P_{max} (MW)	2730	1840	1740	700	594	528	306	208
P_{out} (MW)	2730	1538	1305	700	594	528	306	208
Q_{max} (MVar)	577.4	612	564	86.2	131.1	247.5	74.3	18.2
Q_{out} (MVar)	132.5	151.6	35.6	4.3	-67.5	-9.4	54.1	18.2
V setting (pu)	1.04	1.03	1.04	1.01	1.0	1.01	1.065	1.06

The study case used in this thesis represents the 2000/2001 peak load condition of the BC Hydro system. This heavy load condition is selected since the main concern of this research is the procurement of reactive power support services. The reactive power support is most needed under such conditions. Therefore, the value of reactive power support service provided by a generator is most prominent under the heavy load conditions.

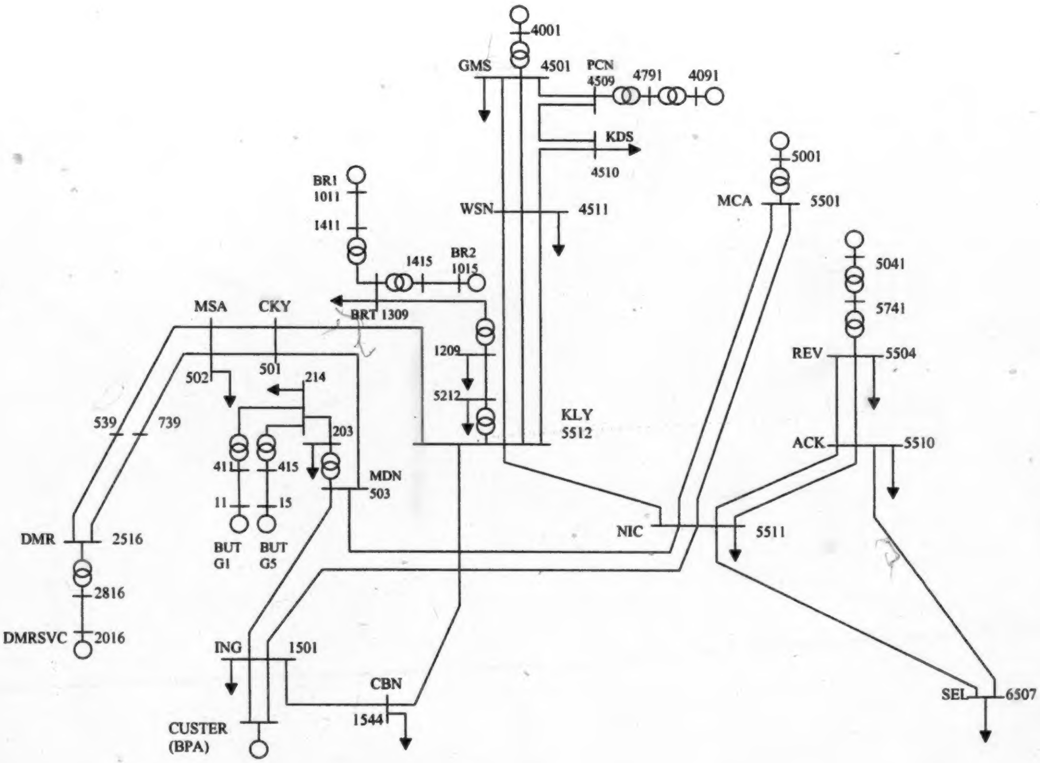


Figure 5.6: Reduced diagram of the BC Hydro system

5.4 Basic Case Study Results

5.4.1 Q_{min} Values:

Typical generators located in different zones are selected as test generators. The Q_{min} values for the selected generators are calculated based on the proportional and OPF generation re-dispatch schemes respectively. When the OPF re-dispatch scheme is used, the formulation of OPF is as follows:

- Objectives: Minimize fuel costs; Minimize slack bus P and Q generation; Optimize the whole system.
- Constraints: 1). Bus voltages within the range of 0.9 ~ 1.1pu; 2). Restrict generators in other areas to participate in the optimization; 3). Keep generator terminal voltages close to those of the base case.

Main results of the Q_{min} values obtained using the OPF and proportional generation re-dispatch approaches are summarized in Table 5.2 and Table 5.3, respectively. In both tables, GMS G1, G6 and G9 represent three generators of the same plant but with different capacities. The Q_{min} results of selected generators are also shown together with their actual reactive power output values in Figure 5.7.

Table 5.2: Q_{min} of typical generators using the OPF re-dispatch scheme

Test Generator	Q *max* (Mvar)	P output[1] (MW)	Q output[1] (MVar)	Ref. Margin (MVar)	Q_{min} (MVar)
BUT	115	160.8	63.89	3762	18.9
BR1	4.55	0.0192	-3.42	3882	1.58
BR2	18.58	0.0193	-0.94	3874	2.06
GMS G1	49.73	261	5.66	3894	359.66
GMS G6	34.2	275	6.94	3898	380.94
GMS G9	100	300	6.1	3904	410.1
MCA	141	435	51.63	3900	225.6
REV G1	153	460	67.91	3912	345.9
KCL	61.87	132	61.87	3908	280.3
SEV	43.71	198	-69.15	3920	430.9
WAH	40	36.62	1.35	3876	1.36
LB1	37	34.08	14.75	3858	-6.25
NWEIPP	32.69	67.5	-4.02	3886	13.98
PCN	21.55	175	-17.36	3478	109.6

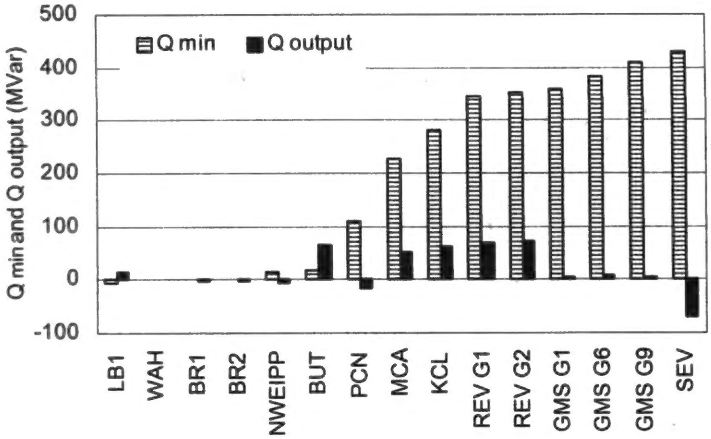
[1] Optimized base case output.

Table 5.3: Q_{min} of typical generators using the proportional re-dispatch scheme

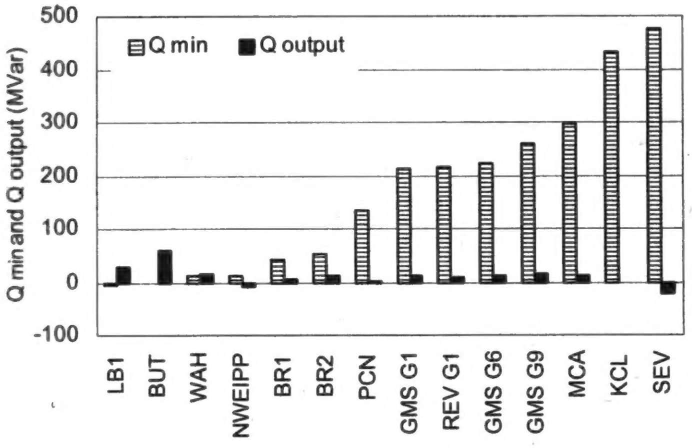
Test Generator	***Q max* (Mvar)**	***P* output[1] (MW)**	***Q* output[1] (MVar)**	**Ref. Margin (MVar)**	**Q_{min} (MVar)**
BUT	115	20	60.9	3560	-0.071
BR1	4.55	52	4.55	3708	42.55
BR2	18.58	76.5	13.54	3700	53.83
GMS G1	49.73	261	12.44	3646	212.44
GMS G6	34.2	275	12.44	3660	223.44
GMS G9	100	300	16.45	3678	258.45
MCA	141	435	11.87	3740	297.87
REV G1	153	158.4	9.15	3708	214.15
KCL	61.87	132	-2.36	3720	433.65
SEV	43.71	198	-22.5	3728	477.26
WAH	40	60	15.25	3678	10.25
LB1	37	55	30.36	3658	-6.64
NWEIPP	32.69	67.5	-6.85	3692	13.15
PCN	21.55	175	1.076	3294	135.1

[1] Original base case output.

The results show that generators GMS, MCA, SEV and KCL have the largest Q_{min} among all the generators. This is due to their large active power output and remote locations. Generators BUT, BR1 and BR2, which are closer to the load center, have relatively small Q_{min}. As a result, reactive power produced by BUT and BR2 can provide more support to the system. The results are consistent with engineering experience. It is noted that most Q_{min} values are significantly larger than the actual Q output. This means that most generators are 'taxing' the system. The reactive power support required is actually provided by the local generators and other reactive power sources, such as capacitors and line charging. The results also show that the OPF and proportional re-dispatch methods yield similar results.



(a). OPF re-dispatch scheme

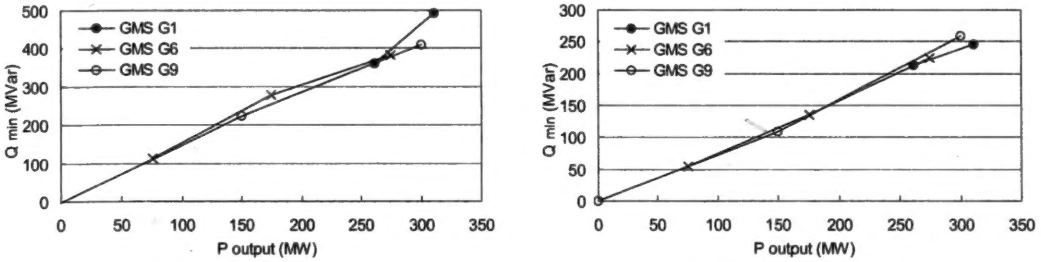


(b). Proportional re-dispatch scheme

Figure 5.7: Comparison of Q_{min} and Q output values

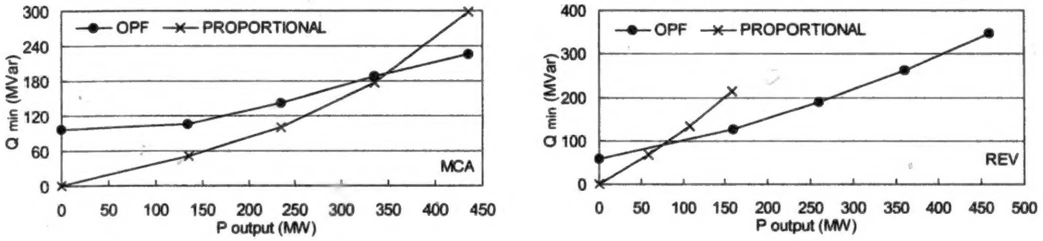
5.4.2 $Q_{min} \sim P_{output}$ Curves:

If the active power output level of the test generator is changed, different Q_{min} values are obtained. The Q_{min} vs. P_{output} curve is obtained by correlating the Q_{min} values with the corresponding active power output level. The obtained curves of key generators are shown in Figure 5.8 and 5.9. Again, both the OPF and proportional re-dispatch schemes have been used. The calculation results of the above plants are summarized and compared in Figure 5.10 (a) and (b).

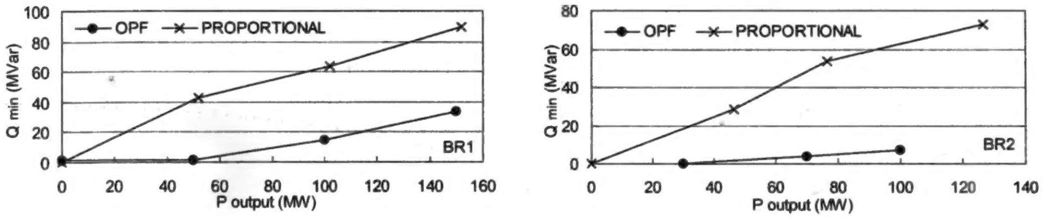


(a). OPF scheme (b). Proportional scheme

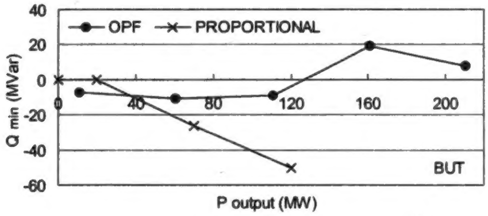
Figure 5.8: $Q_{min} \sim P_{output}$ curves of Plant GMS



(a). MCA (b). REV

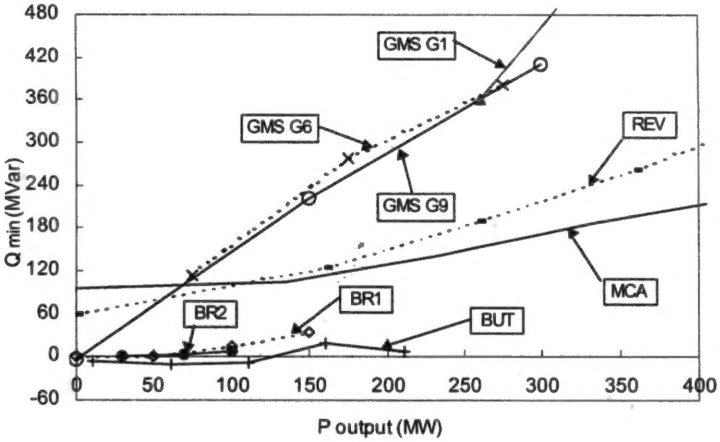


(c). BR1 (d). BR2

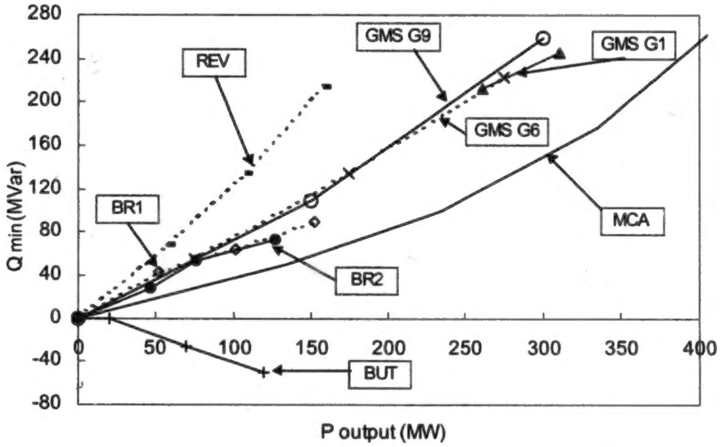


(e). BUT

Figure 5.9: $Q_{min} \sim P_{output}$ curves of Plants MCA, REV, BR1, BR2 and BUT



(a). OPF re-dispatch scheme

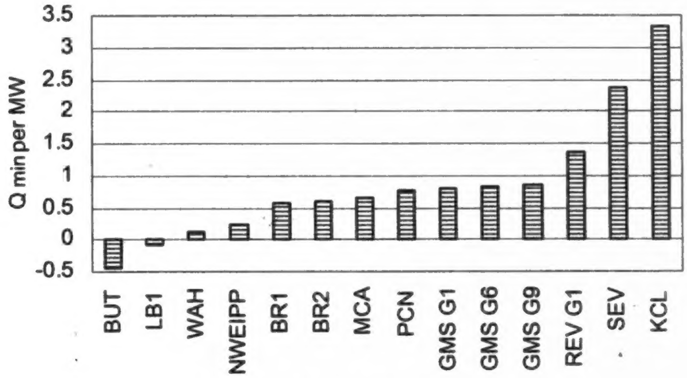


(b). Proportional re-dispatch scheme

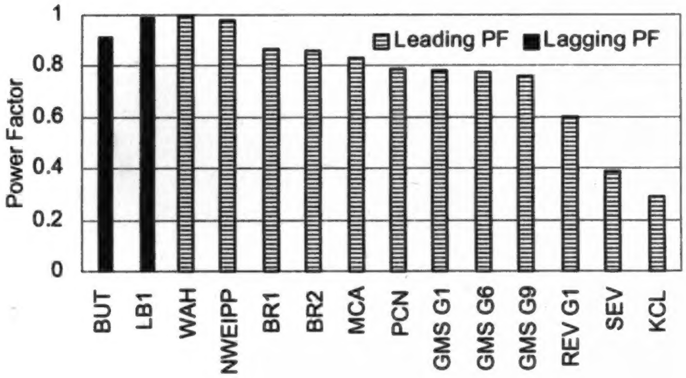
Figure 5.10: Comparison of Q_{min} vs. P_{output} curves of different generators

In Figure 5.10, most of the curves have a positive slope. The negative slope of BUT implies if more active power is produced by BUT instead of by other remote generators, the system margin would increase. It agrees with the fact that increasing the active power output of the generators close to load center always improves the system stability margin. Figure 5.10 also shows that the $Q_{min} \sim P_{output}$ curves are almost linear for all test generators. It is, therefore, reasonable to divide the Q_{min} values by their corresponding P

output levels. The results represent the required 'unit' Q_{min} for 1MW active power output of the generators. Another way to view these results is the maximum allowed power factor for the generators, which is shown in Figure 5.11.



(a). Q_{min} per MW output

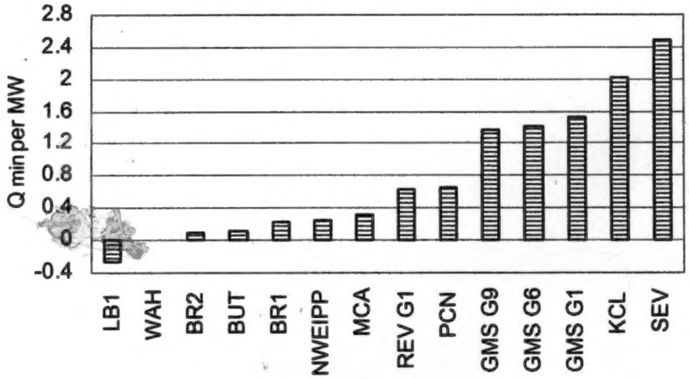


(b). Power Factor Comparison

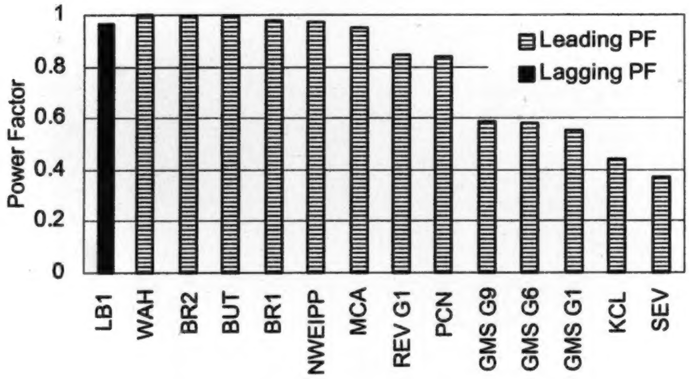
Figure 5.11: Comparison of different generators — Proportional approach

Figure 5.11 (a) shows that SEV and KCL have the largest per MW Q_{min} values, while LB1, WAH and BUT have the smallest. Compared with BR1 and BR2, GMS has a larger unit Q_{min}, which is consistent with engineering experience. The power factor chart shows that most remote generators have to generate a large amount of reactive power. This requirement may render their active power selling ventures unattractive. However, such generators can provide reactive power support in an alternative form such as adding shunt capacitors to alleviate the power factor requirement. It is possible that a generator's terminal voltage could be unrealistically high if it does output more reactive power than Q_{min}. Such cases could imply that the generator is unable to transmit its own active power without the reactive power support from the system.

The above results are based on proportional generation re-dispatch scheme. The OPF based dispatch scheme has also been tested. The results are shown in Figure 5.12. It is found that when Q_{min} per MW values are compared, the two approaches show an overall consistency.



(a). Q_{min} per MW output



(b). Power Factor Comparison

Figure 5.12: Comparison of different generators — OPF approach

The above case studies have also been carried out in a larger BC Hydro system case. The case includes more areas in the West Coast Canada-US-Mexico system. Area 1 (BC Hydro) of this case is the same as the case used in previous studies. There are 8309 buses, 23 areas, 9 zones, 1290 power plants and 10970 branches. The Q_{min} results obtained from this larger case are consistent with those presented in this section.

5.5 Sensitivity Study Results

Sensitivity studies are carried out to test the performance characteristics of the proposed method. Main findings are presented as follows:

5.5.1 Impact of Active Power Output on Q_{min}

The three different-sized generators at bus GMS provide a good example to illustrate the impact of active power output on Q_{min}. As shown in Figure 5.7(b), the generator with smallest P output (261MW), G1, has the smallest Q_{min} value (212.2MVar), while the largest Q_{min} (258.5MVar) comes from the largest generator G9 (300MW). This again verifies the previous conclusion in this thesis that a larger Q_{min} is expected for a larger active power transmission. Furthermore, when the per MW Q_{min} is calculated and compared, there is no difference among the three generators, due to their identical location in the system.

5.5.2 Impact of Plant Location on Q_{min}

Location of a generating plant is an important factor affecting the value of Q_{min}. Since generators are widely distributed in a system, it is expected that the same amount of

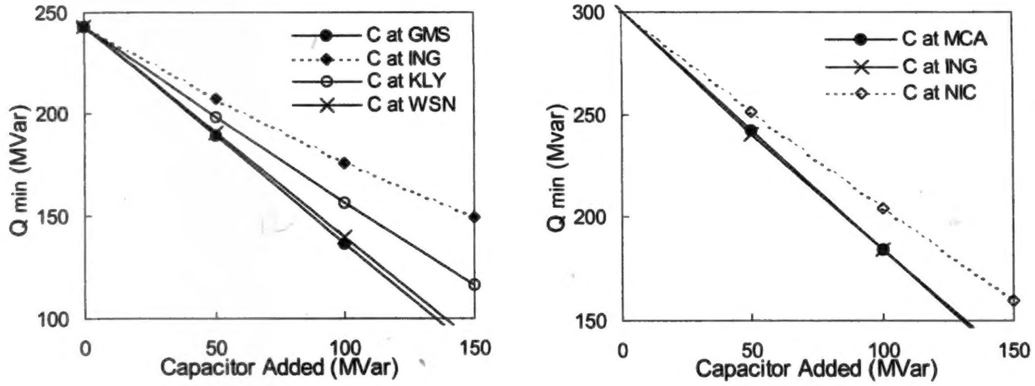
P output at different locations would require different Q_{min}. In this study, the impact of generator locations is shown in Figure 5.11 (a). For the test system, remote plants such as SEV, KCL, GMS and REV have larger Q_{min} compared with the local plants such as BUT and BR1.

5.5.3 Impact of Shunt Compensation on Q_{min}

Results shown in previous sections reveal that many remote generators need to output more reactive power to avoid a negative impact on the system margin. Besides increasing the generator terminal voltage, adding shunt compensation can be an alternative to reduce the Q_{min} level. In this study, shunt capacitors are added at the generator terminals, load centers and other locations respectively to assess their impacts on Q_{min}.

1). Numerical Study Results

GMS and MCA are used as test generators for this study. The results are shown in Figure 5.13. It is found that there is a linear relationship between the capacitor size and Q_{min}, if the capacitors are added at the generator terminal. This agrees with intuition. The amount of reduction on Q_{min} is essentially equal to the size of the capacitor added. The other curves in the figure show the results of adding the capacitor at other locations, such as at the load center (ING). It can be seen that as the size of capacitor increases, the Q_{min} also decreases. However, adding capacitors at the load center does not necessarily yield the largest reduction on the Q_{min} value. This phenomenon can be understood as follows: the reactive power output of the capacitor at the load center also supports the active power transmission of other generators in addition to the test generator. The following analytical studies explain the observations further.



(a). Q_{min} for GMS (b). Q_{min} for MCA

Figure 5.13: Impact of adding capacitor at different locations

2). Analytical Study

The simplified system in Figure 5.14 is used to illustrate the effectiveness of the shunt capacitor. In this simplified system, Q_c represents the amount of reactive power supplied by a capacitor. The location of the capacitor is changed to analyze its impact on the Q_{min} of the generator. The Q_{min} should be the value at which the system provides no reactive support to help the transmission of the active power, i.e. $Q_{sys} = 0$. All resistance has been neglected. The following cases are analyzed, respectively.

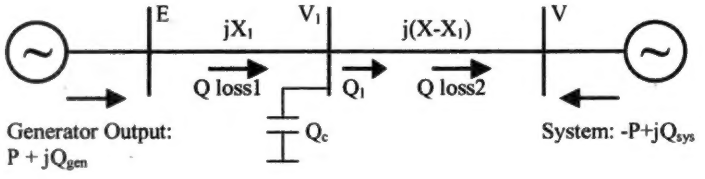


Figure 5.14: Sample system with capacitor at different locations

- *Case 1: Without capacitor*:

Equation (5.1) can be obtained for the system in Figure 5.14 when there exists no capacitor. The Q_{min} can be calculated when Q_{sys} is set to zero, as shown in Equation (5.2).

$$\begin{aligned} Q_{gen} &= \frac{E^2}{X} - \sqrt{(\frac{EV}{X})^2 - P^2} \\ Q_{sys} &= \frac{V^2}{X} - \sqrt{(\frac{EV}{X})^2 - P^2} \\ Q_{sys} &= Q_{gen} + \frac{V^2}{X} - \frac{E^2}{X} \end{aligned} \tag{5.1}$$

$$Q^0_{\min} = \frac{P^2}{V^2} X \tag{5.2}$$

- *Case 2: Adding a capacitor to the generator terminal*:

Similar to Equations in (5.1), the equations shown in (5.3) can be obtained in this case. Correspondingly, Q_{min} is derived in Equation (5.4):

$$\begin{aligned} Q_{gen} + Q_c &= \frac{E^2}{X} - \sqrt{(\frac{EV}{X})^2 - P^2} \\ Q_{sys} &= \frac{V^2}{X} - \sqrt{(\frac{EV}{X})^2 - P^2} \\ Q_{sys} &= Q_{gen} + \frac{V^2}{X} - \frac{E^2}{X} + Q_c \end{aligned} \tag{5.3}$$

$$Q^1_{\min} = \frac{P^2}{V^2} X - Q_c \tag{5.4}$$

Obviously, the Q_{min} difference between no capacitor and adding a capacitor to the generator terminal is Q_c, i.e.:

$$Q^{1}_{\min} = Q^{0}_{\min} - Q_c \tag{5.5}$$

- *Case 3: Adding a capacitor to the system/load side*:

Similarly, the following equations in (5.6) can be obtained and Q_{min} is calculated from Equation (5.7):

$$\begin{aligned} Q_{gen} &= \frac{E^2}{X} - \sqrt{(\frac{EV}{X})^2 - P^2} \\ Q_{sys} + Q_c &= \frac{V^2}{X} - \sqrt{(\frac{EV}{X})^2 - P^2} \\ Q_{sys} &= Q_{gen} + \frac{V^2}{X} - \frac{E^2}{X} - Q_c \end{aligned} \tag{5.6}$$

$$Q^{2}_{\min} = \frac{P^2}{V^2} X - Q_c + \frac{Q_c^2}{V^2} X \tag{5.7}$$

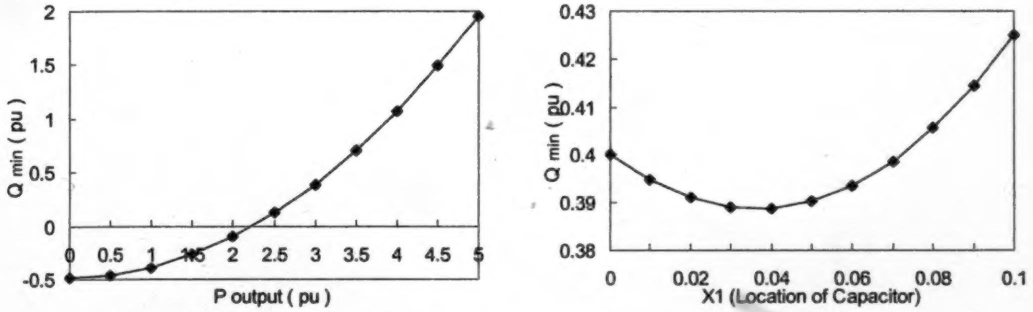
Comparing Equation (5.7) with (5.4), it is obvious that adding a capacitor to the system or load side will result in a larger Q_{min} than adding the same capacitor to the generator terminal.

- *Case 4: Adding a capacitor in the middle of the line*:

Similar approach has been adopted. When $Q_{sys} = 0$, the Q_{min} can be derived from the equation below:

$$Q_{\min} = \frac{P^2}{V^2}(X - X_1) - Q_c + \frac{P^2V^2 + (\frac{P^2}{V}(X - X_1) - Q_cV)^2}{V^4 + P^2(X - X_1)^2} X_1 \tag{5.8}$$

Equation (5.8) shows that Q_{min} is a function of active power P, line reactance X_l, voltage V and the size of the capacitor Q_c. Some numerical data are used to illustrate the impact of the above factors on Q_{min}. The following parameters are used for the system in Figure 5.14: P=3pu, X=0.1pu, X_l=0.05pu, V=1.0pu and Q_c=0.5pu (base: 100MVA). Figure 5.15(a) shows that, when the active power P increases, Q_{min} also increases accordingly. It is interesting to notice that Q_{min} is negative when P=0. This means that the capacitor provides support to the system. With the help of the capacitor, the generator could transmit the active power up to 2.2 p.u. with zero Q_{min} requirement.



(a). $Q_{min} \sim P$ curve:

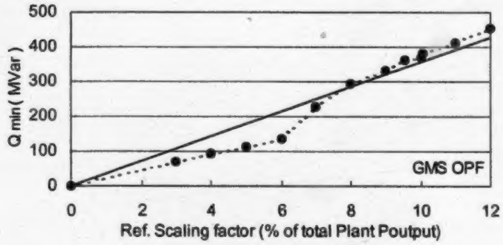
(b). $Q_{min} \sim X_l$ curve

Figure 5.15: Impacts of factors on Q_{min} with capacitor in the middle of line (Base: 100MVA)

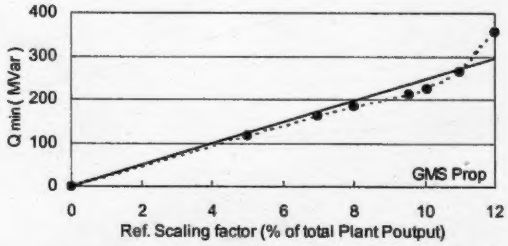
Figure 5.15 (b) confirms the previous results --- when the capacitor is at the system side, Q_{min} is larger than when capacitor is at the generator terminal. However, neither of the two cases generates the smallest Q_{min}. The best location that results in the smallest Q_{min} is somewhere between the generator terminal and the system side, which yields the least total reactive power loss. The selection of optimal capacitor locations to reduce Q_{min} requirement on generators is an interesting research problem. Without any method for location selection at present, it is recommended to add capacitors at the generator terminal to lower the Q_{min} requirement.

5.5.4 Impact of Reference Case Selection

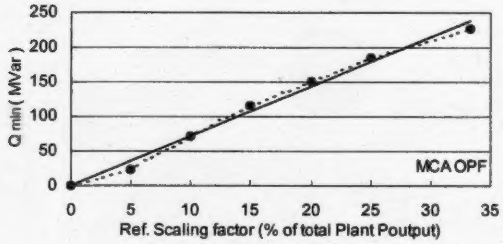
The reference case plays an important role for Q_{min} calculation. Ideally, the reference case should be formed by taking all generators in a plant offline. This is not feasible for large generating plants, since they cannot shutdown completely due to the need to meet load demand. In this research, the problem is solved by scaling down the plant's active power output. The corresponding scaled-down case is the reference case. The effect of the scaling factor on the Q_{min} results is investigated by calculating and recording the Q_{min} value for each scaling factor. Case studies were conducted for large plants such as GMS, MCA, KCL etc. The results are shown in Figure 5.16, where the real Q_{min} calculation data are shown by dotted lines and the solid straight lines represent the first order polynomials that fitted the data.



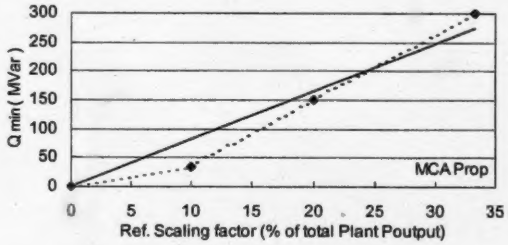
(a). GMS with OPF scheme



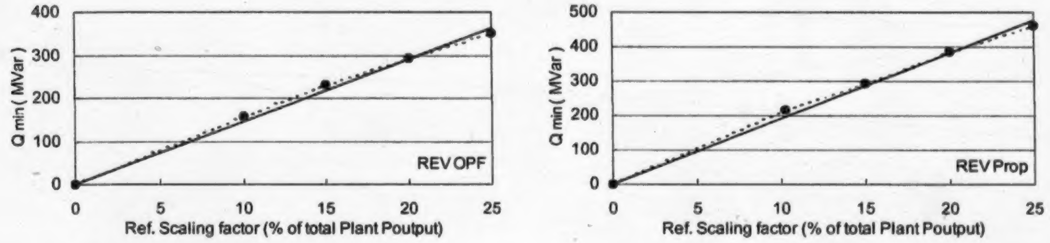
(b). GMS with proportional scheme



(c). MCA with OPF scheme

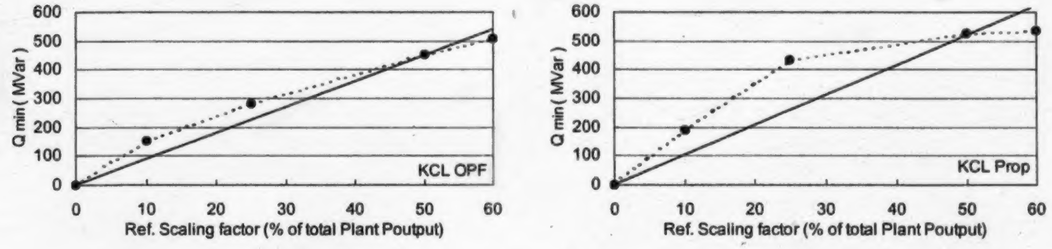


(d). MCA with proportional scheme



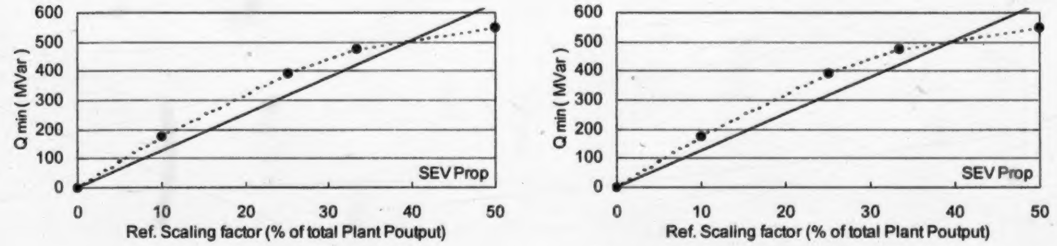
(e). REV with OPF scheme

(f). REV with proportional scheme



(g). KCL with OPF scheme

(h). KCL with proportional scheme



(i). SEV with OPF scheme

(j). SEV with proportional scheme

Figure 5.16: Q_{min} ~ Scaling factor curves

Each scaling factor in the above figures represents a certain amount of P output, that is, the Q_{min} ~ Scaling factor curves are also Q_{min} ~ P output curves. The above results show a relatively linear relationship between Q_{min} and the scaling percentage. Therefore, it is possible to use extrapolation to find out the Q_{min} for the P output of the whole plant.

5.6 Analysis of Closely Coupled Power Plants

The phenomenon of closely coupled power plants is observed during the Q_{min} calculation since it's hard to find Q_{min} of the generators at PCN (Please see Figure 5.6). PCN is a power plant which has a very short transmission line connected to GMS, one of the most important power plants in BC Hydro system. Detailed studies show that increasing Q output of PCN has little impact on system margin. This phenomenon is illustrated by the following simplified system. The simplified system has two closely connected generators providing power to the same load center. G1 represents GMS and G2 represents PCN. The system parameters are as follows (Base: 100MVA):

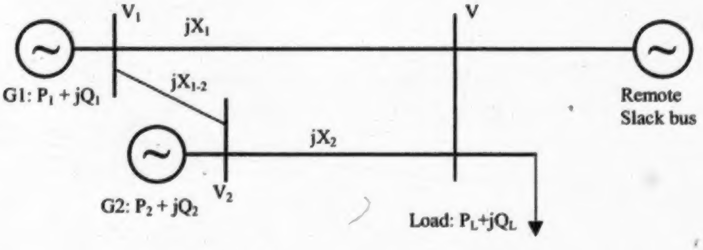


Figure 5.17: Simplified system for PCN problem illustration

— Line impedance: $X_1 = 0.1$pu, $X_2 = 0.2$pu, $X_{1\text{-}2} = 0.01$pu

— Load: 180+j90 MW

— Slack bus voltage setting: $V = 1.03$pu; G2 voltage setting: $V_2 = 1.005$pu

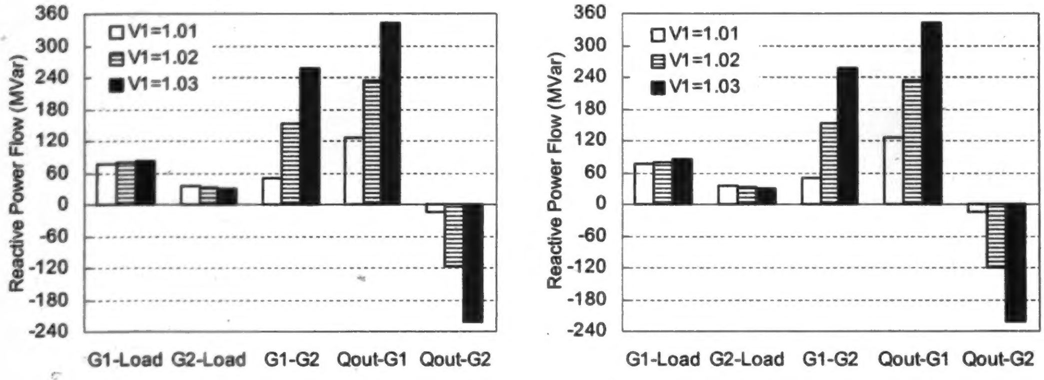
Two scenarios are considered to test the impact of plant capacity on system margin:

— Scenario 1: $P_1 = 40$MW, $P_2 = 140$MW

— Scenario 2: $P_1 = 140$MW, $P_2 = 40$MW

Numerical study is done by changing the voltage setting of G1. The load flow results and system margin are calculated and recorded. *Q* output of both generators is shown in Figure 5.18.

It can be observed that *Q* output of G1 increases greatly when the voltage setting of G1 is increased. Most of the increased *Q* output of G1 goes to G2 through the short connection line. The reactive power goes to load changes only slightly. As a result, the action of increasing G1's *Q* output has little impact on the system margin. These numerical results show that:



(a). Scenario 1 (b). Scenario 2

Figure 5.18: Reactive power output and flow changes

- The phenomenon of closely coupled power plants was only encountered once in the case studies of this research. It is not related to the meshed network or any modeling problem. This phenomenon could occur for generators connected very close to each other. One generator could dominate the voltage profile and reactive power flow pattern in the area.
- Changes made by other non-dominant generators have little impact on the reactive

power flow pattern. As a result, the system margin is not sensitive to those changes.

- Voltage setting is also a factor in the above phenomenon. The generator with higher terminal voltage produces little impact on system margin. Therefore, it is very possible that the Q_{min} of the generator with higher voltage setting can not be found.

The proposed solution in this thesis for the above problem is to treat the closely coupled power plants or generators as one group. This will help to avoid the 'coupling effect' between the two plants or generators.

5.7 Case Study Results of the Alberta System

The proposed Q_{min} method is further tested on the actual Alberta system. Similar to the test on the BC Hydro system, the amount of minimum reactive power support requirement for the key generators in the Alberta system is obtained and summarized in this section. Some sensitivity studies are also conducted to test the robustness of the method.

5.7.1 The Alberta Interconnected Electric System

The basic system data of the Alberta Interconnected Electric System were obtained from ESBI Alberta Ltd., who functioned as the Alberta Transmission Administrator when the case was created in 1999. This system is mainly composed of 30 zones and 23 areas, with 1755 buses, 114 plants, 1097 loads and 1956 branches. The total generation is 7396MW and the total load is 6827MW. Area 6(Calgary), 9(Edmonton) and 3(N. Cent) are the main load centers, while area 3(N. Cent), 9(Edmonton) and 14(Drumheller) are where the main power generation located. Figure 5.19 shows the main structure of the Alberta system and its key plants.

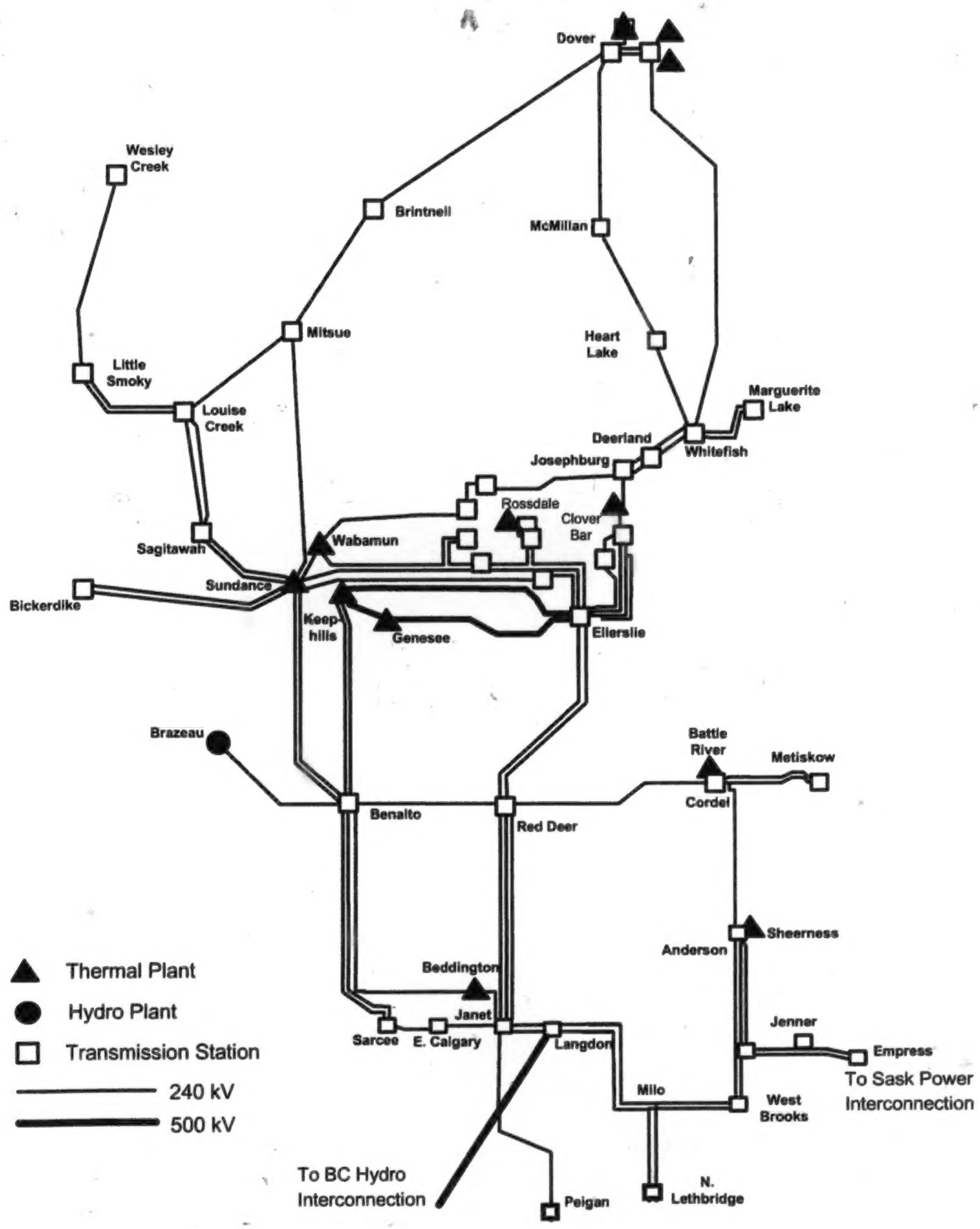


Figure 5.19: Main structure of the Alberta system

In the Alberta system, power transmission is mainly through 240kV transmission lines. Plants such as Keephills and Sundance provide power supply to the Calgary area via Benalto through two 240kV transmission lines each. Other plants such as Genesee, also supply power to Red Deer through two 240kV transmission lines. The power is then transmitted to Calgary via Janet. The main geographic structure and areas of the Alberta system are shown in Appendix A. Table 5.4 lists the data of important generators.

Table 5.4: Key generator data of the Alberta system

Machine Name	Bus No.	Voltage Level (kV)	*Pmax* (MW)	*Pout* (MW)	*Qmax* (MVar)	*Qout* (MVar)	*V* setting
Genesee #1	491	240	401	386	116	94	1.0408
Keephills #2	424	240	383	383	120	116	1.066
Sheerness 1	1482	240	383	380	200	61.7	1.0844
Bat #5	1497	240	371	370	110	1.7	1.0699
Sundance#6	350	240	366	366	241	46.5	1.0145
Sundance#4	342	240	355	355	238	81.5	1.0293
Sundance#1	129	240	280	280	140	34.4	1.0124
Wabamun#4	146	240	280	280	140	78.9	1.0524
Braz#2	154	240	192	30	70	36.8	1.0428
CBAR 3	495	240	172	158	55	55	1.0647
BAT #3	1495	144	148	148	40	20.9	1.0794
HR MILNR	1148	144	145	145	40	13.9	1.0689
Wabamun#2	140	138	64	64	33	22.5	1.023
SYNC G6	19210	72	63	40	40	-1.1	1.0347

5.7.2 Basic Case Study Results

1). Q_{min} Values:

Typical generators at different locations in Alberta are selected as test generators. The Q_{min} for the selected generators are calculated based on proportional generation re-dispatch scheme. Main results of the Q_{min} values obtained are summarized in Table 5.5.

In this table, Sundance #6, #4 and #1 represent three generators in the same plant but with different capacities. The Q_{min} results of selected generators are also shown together with their actual reactive power output values in Figure 5.20.

Table 5.5: Q_{min} of typical generators

Test Generator	**Bus No.**	***Q max* (Mvar)**	***P* output[1] (MW)**	***Q* output[1] (MVar)**	**Ref. Margin (MVar)**	**Q_{min} (MVar)**
Genesee #1	491	116	386	94	2550	**267.9**
Keephills #2	424	120	383	116	2530	**231.0**
Sheerness 1	1482	200	380	61.7	2082	**-92.8**
Bat #5	1497	110	370	1.71	2288	**-55.3**
Sundance #6	350	241	366	46.5	2356	**186.5**
Sundance #4	342	238	355	81.5	2308	**176.5**
Sundance #1	129	140	280	34.4	2312	**133.4**
Wabamun #4	146	140	280	78.9	2416	**139.9**
Braz#2	154	70	30	36.8	2338	**-9.2**
CBAR 3	495	55	158	55	2396	**20**
BAT #3	1495	40	148	20.9	2340	**-54.1**
HR MILNR	1148	40	145	13.9	2372	**-21.1**
Wabamun #2	140	33	64	22.5	2390	**-8.5**
SYNC G6	19210	40	40	-1.1	2250	**21.9**

The results show that the generators at Genesee, Keephills and Sundance have the largest Q_{min} among all the generators. This is due to their large active power output and remote locations to the load center, which is the Calgary area. These generators are located in the Edmonton area. The major portion of the power they produce is transmitted to Calgary through 240kV transmission lines over long distance. For these generators, the Q_{min} values are significantly larger than their actual reactive power output. This means that these generators are drawing reactive power support from the system. On the contrary, generators such as those at Sheerness and Battle River have small, even negative Q_{min} values. This is due to the fact that the power produced is consumed locally.

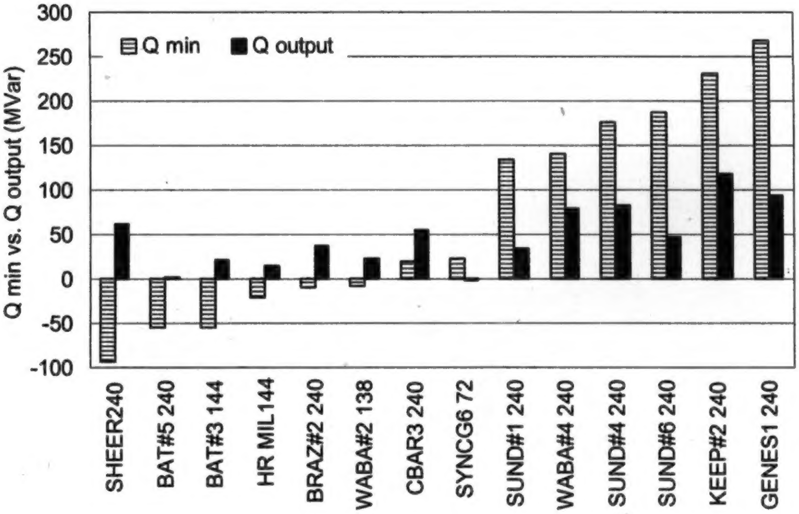
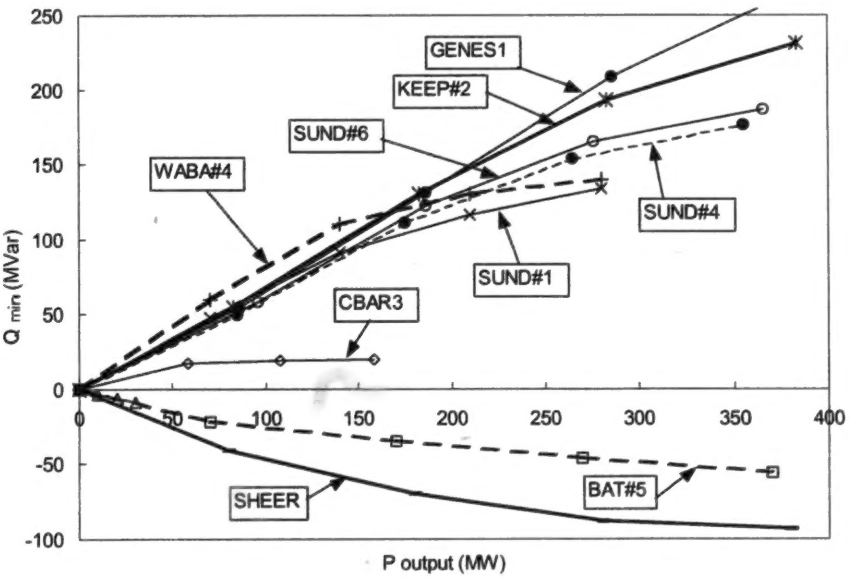


Figure 5.20: Q_{min} and Q output value comparison

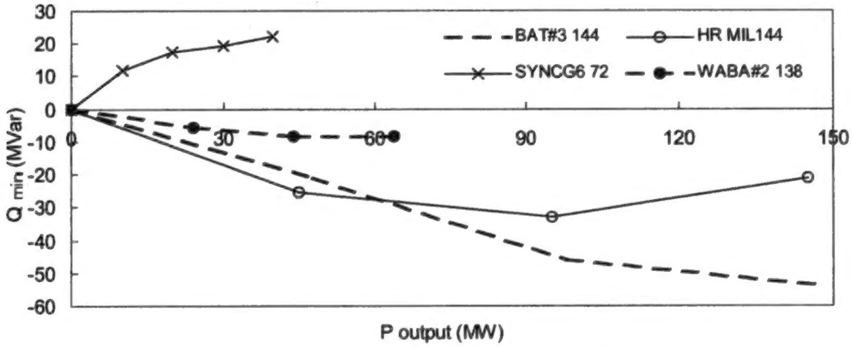
2). $Q_{min} \sim P_{output}$ Curves:

If the active power output level of the test generator is changed, different Q_{min} values are obtained. The Q_{min} vs. P_{output} curve is obtained by correlating the Q_{min} values with the corresponding active power output levels. The obtained curves of key generators are shown in Figure 5.21 (a) and (b).

In Figure 5.21(a), most of the curves have a positive slope. The negative slope of Sheerness and Bat #5 implies if more active power is produced by these generators instead of other remote generators, the system margin would increase. It agrees with engineering experience that increasing the active power output of the generators close to load center always improves the system stability margin. Figure 5.21(b) shows that generators transmitting power through lower voltage levels require relatively smaller Q_{min} values. This is because usually these generators are satisfying local load demands. It can also be seen from Figure 5.21(a) and (b) that the $Q_{min} \sim P_{output}$ curves are almost linear for most of the test generators. Therefore it is reasonable to divide the Q_{min} values by their corresponding P output levels, similar to the process done for the BC Hydro system. The results represent the required 'unit' Q_{min} for 1MW active power output of the generators. Another way to view these results is the maximum allowed power factor for the generators. The results are shown in Figure 5.22(a) and (b).

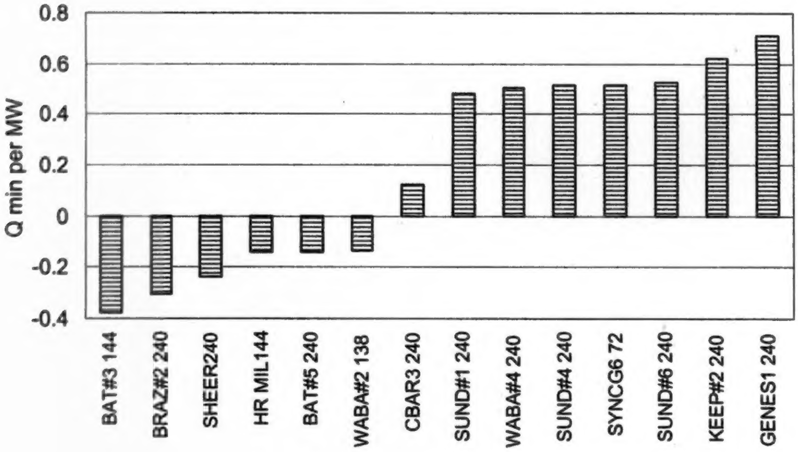


(a). Generators connected to 240kV voltage level.

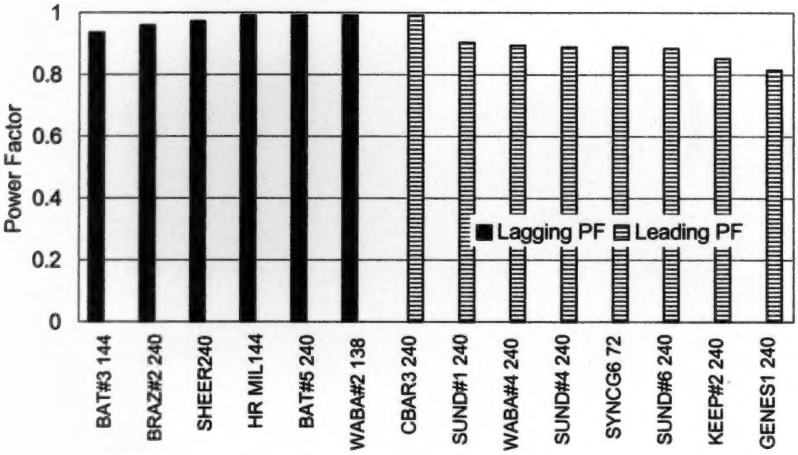


(b). Generators connected to 138kV or 69kV voltage level.

Figure 5.21: Comparison of Q_{min} vs. P_{output} curves



(a). Q_{min} per MW output



(b). Power Factor Comparison

Figure 5.22: Comparison of different generators

Figure 5.22(a) shows that the generators at Genesee, Sundance and Keephills have the largest per MW Q_{min} values, while those at Sheerness and Battle River (Bat #5 and #3) have the smallest. Compared with generators at Sundance, generators at Wabamun have a smaller unit Q_{min}, although their physical locations are close. This is because the power output of Wabamum is mainly used to supply local load demands in the Edmonton Area. This phenomenon is further analyzed in the sensitivity studies. The power factor chart shows that most remote generators have to generate a large amount of reactive power. This requirement may render their active power selling ventures unattractive. Such generators may choose to provide reactive power support in an alternative form such as adding shunt capacitors to alleviate the power factor requirement.

5.7.3 Sensitivity Study Results

Sensitivity studies are carried out on the Alberta system in the following aspects. Main findings are presented as follows:

1). Impact of Active Power Output on Q_{min}

The three different-sized generators at Sundance are used as an example to illustrate the impact of active power output on Q_{min}. As shown in Table 5.5, generator Sundance #1 has the smallest active power output (280MW). It also has the smallest Q_{min} value (133.44MVar). The other two types of generators, Sundance #4 and #6, have larger active power output of 355MW and 366MW, respectively. Correspondingly, their Q_{min} values (176.5MVar and 186.5MVar, respectively) are larger than that of Sundance #1. This again verifies the previous conclusion that a larger Q_{min} is expected for a larger active power transmission.

Furthermore, when the per MW Q_{min} is calculated and compared, there is no big difference among the three generators. The $Q_{min} \sim P_{output}$ curves for the three different-sized generators are almost identical, as shown in Figure 5.23 below.

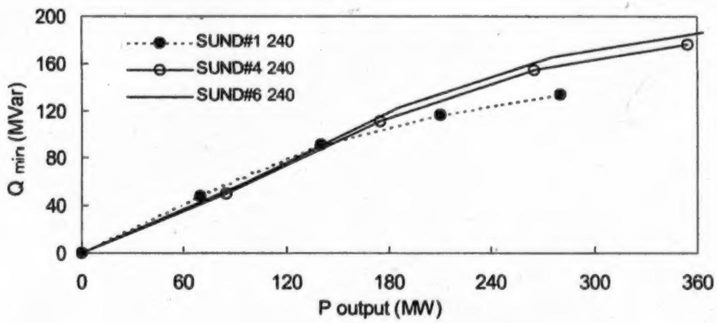


Figure 5.23: Q_{min} comparison for generators at Sundance

2). Impact of Plant Location on Q_{min}

Location of a power plant is an important factor affecting the value of Q_{min}. Since generators are widely distributed in a system, one would expect that the same amount of P output at different locations would require different amount of Q_{min}. Previous results show that generators close to load centers have a smaller even negative Q_{min} requirement; while generators transmitting power through long distance have larger Q_{min} needs. In the Alberta system, generators belonging to the first situation include those at Sheerness and Battle River. Sundance, Genesee and Keephills have generators that belong to the latter case. In this study, these generators are further analyzed to illustrate their differences in Q_{min} requirements.

- *Generators at Sheerness and Battle River*:

Sheerness and Battle River are in the middle east of the Alberta system. The Battle River area is directly east of Red Deer, while Sheerness is further to the south. The load flow patterns of these areas are recorded to analyze which areas are actually supplied by these generators. The load flow patterns are briefly shown in Figure 5.24 and 5.25.

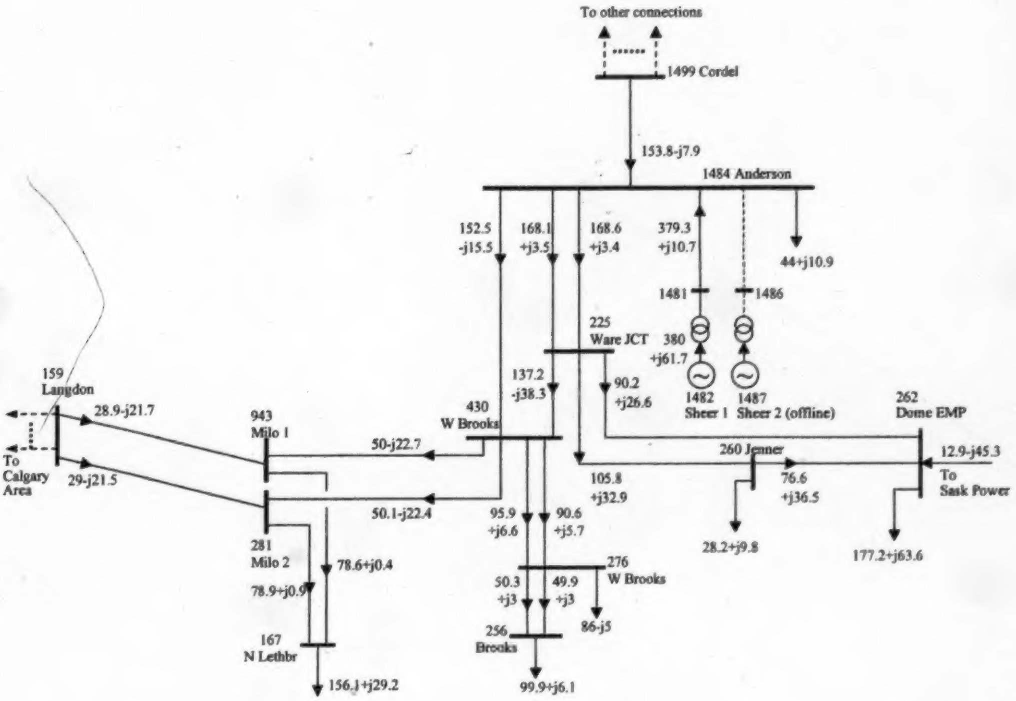


Figure 5.24: Load flow of the Sheerness area

Figure 5.24 shows that power generated from the Sheerness Plant is mainly used for local load demands. A portion of the power is transmitted via Ware JCT to the east. Another portion of the power supplies loads in the south including the W. Brooks area and N. Lethbridge (N. Lethbr.) area. There is no power entering the backbone grid of the Alberta system, i.e. the Edmonton to Calgary transmission system. On the contrary, power is transmitted to this area from Calgary in the west and from Sask Power in the east. Some power is also supplied from the North Battle River area. Similarly, Figure 5.25 shows that power produced from the Battle River #5 Generator is also used for local load demands. The power is transmitted to Metiskow in the east, to Anderson in the south and to Red Deer in the west. Red Deer is the transfer substation in the Alberta backbone subsystem. Figure 5.25 shows that large amount of power flows from the Edmonton area to the Calgary area through this substation. Battle River #5 generator only contributes a very small portion of power to the Alberta backbone.

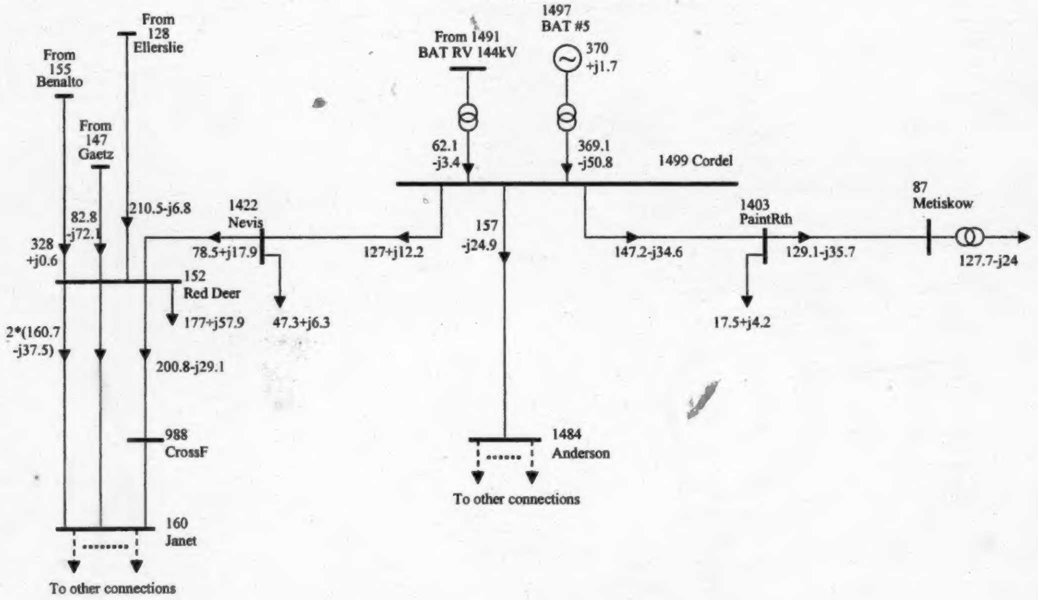


Figure 5.25: Load flow of the Battle River area

The above load flow analysis illustrates that power generated at Plant Sheerness and Battle River is mainly used for satisfying local load demands. In other words, these generators are important local generators close to the 'load centers'. As a result, it is desirable to have these generators output more power to improve the system security level. Therefore, the Q_{min} of these generators have negative values.

- *Generators at Sundance, Keephills and Genesee:*

Similarly to the above analysis, the load flow pattern of Genesee, Sundance and Keephills area is shown in Figure 5.26. These three plants reside close to each other east of Edmonton and produce the major portion of generation in the province.

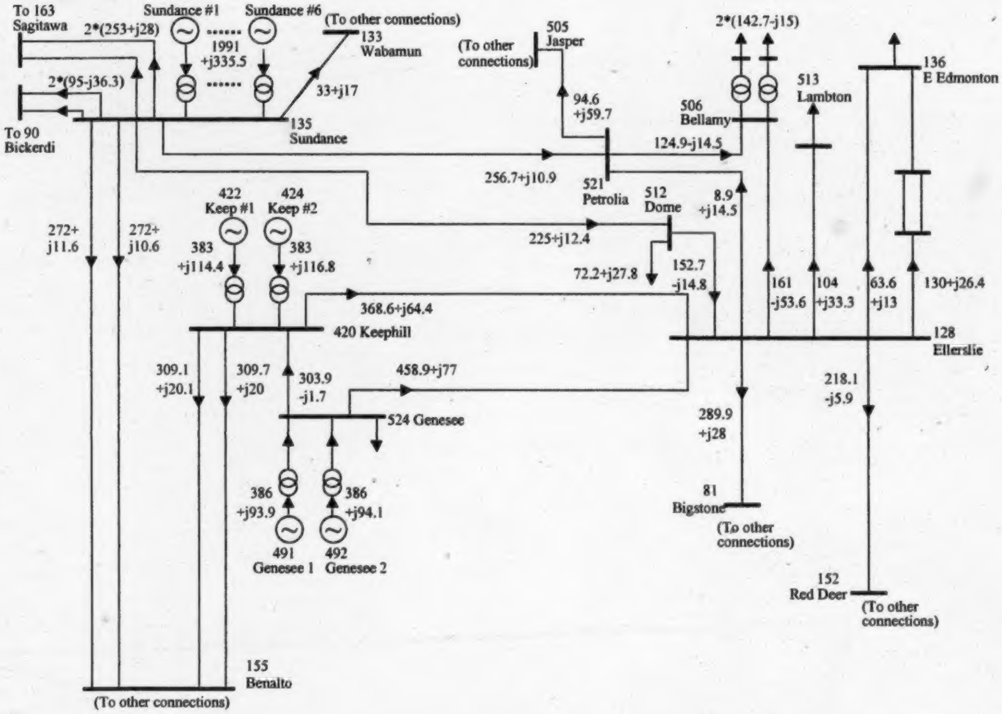


Figure 5.26: Load flow of the East Edmonton area

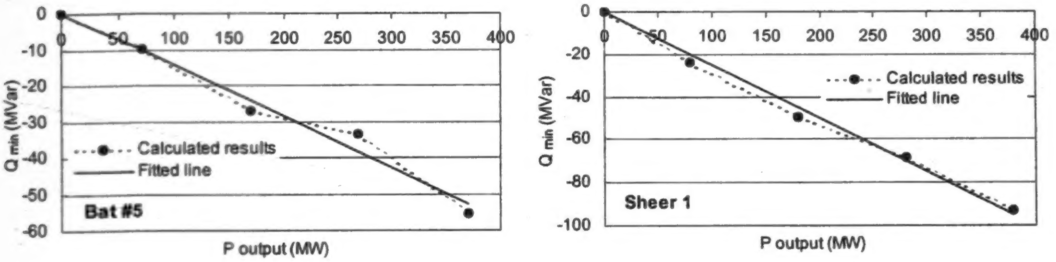
Figure 5.26 shows, that a large portion of the power produced is transmitted to the south --- the Calgary area via different paths. Keephills and Sundance have direct transmission lines connected to Benalto, a transfer substation between the Edmonton area and the Calgary area. The power is then transmitted to Sarcee and Janet in the Calgary area. Another path going south is through Ellerslie, Red Deer and Bigstone. The power is then transmitted down to the south from Red Deer. This load flow pattern clearly shows one of the important features of the Alberta system: a large amount of power is generated in the north (Edmonton area). The power is then transmitted to the south (Calgary area) by 240kV lines over long distance. The large power output levels and the long distances from the load center, determine that these generators require large reactive power support from the system. As a result, large Q_{min} results are expected.

The same analysis can also be carried out to the plant at Lake Wabamun, which is also close to the Edmonton area. It can be shown that the power from this plant is mainly used for local loads in the Edmonton area. Therefore, the generators at Wabamun have smaller Q_{min} values compared to the generators at Genesee, Sundance and Keephills.

3). Impact of Reference Case Selection

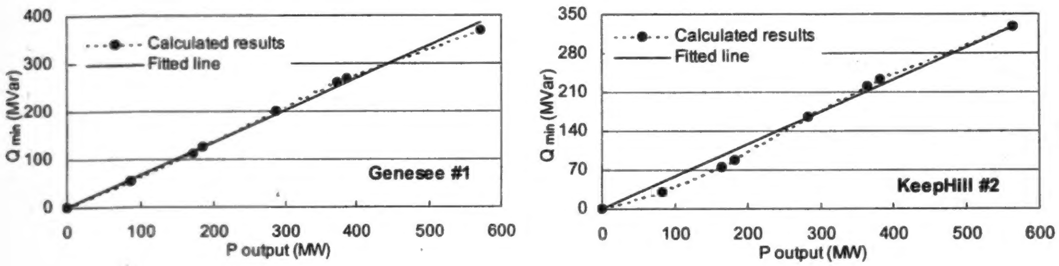
Similar to the study on the BC Hydro system, the active power of a whole plant is scaled down to form the reference case. The effect of the scaling factor on the Q_{min} results is investigated. Since each scaling factor represents a certain level of active power output of the plant, the Q_{min} value for each scaling factor is recorded and correlated with the corresponding equivalent active power output. Case studies were conducted for large plants such as Genesee, Sundance, Keephills etc. The results are shown in Figure 5.27. The Q_{min} calculation data are shown by dotted lines and the solid straight lines represent the first order polynomials that fitted the data.

The above results show a relatively linear relationship between Q_{min} and the active power output after scaling down. Therefore, it is possible to use extrapolation to find out the Q_{min} for the active power output of the whole plant.



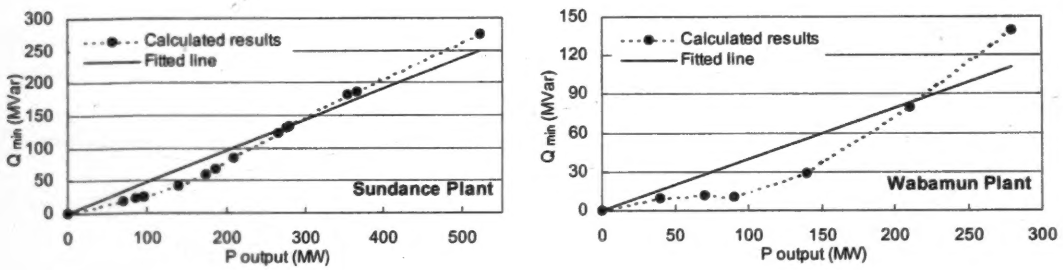
(a). Test plant: Battle River

(b). Test plant: Sheerness



(c). Test plant: Genesee

(d). Test plant: KeepHills



(e). Test plant: Sundance

(f). Test plant: Wabamun

Figure 5.27: Q_{min} curves using scaling factors

5.8 Summary

The proposed Q_{min} method is further improved in this chapter by considering practical constraints. This final generalized method is tested in actual large systems and promising test results are found. The following conclusions are reached:

- Extensive case study results have confirmed the validity of the proposed Q_{min} concept and demonstrated its usefulness.
- Case study results show that Q_{min} is a function of several factors, such as active power P, line reactance X etc. A higher Q_{min} is required when more active power is transmitted or the generator is far away from the load center.
- When calculating Q_{min} values, both proportional and OPF approaches can be used in generation re-dispatch. The results are consistent and comparable as long as the same approach is used for all generators.
- Study results show that the generalized Q_{min} method is suitable for large-scale power systems. The method has been successfully used in the actual BC Hydro system and the Alberta Interconnected Electric system. The robustness of the method has been found to be very good.
- Sensitivity studies show the impacts of possible factors on the Q_{min} calculation. It is recommended to add capacitors at the generator terminal to reduce the required Q_{min} level. It is also recommended to use scaling factors for large power plants in the Q_{min} calculation.

Chapter 6

Application of Q_{min} for the Procurement of Reactive Power Support Services

The minimum reactive power requirement (Q_{min}) method and case study results are introduced in Chapters 4 and 5. It is shown that Q_{min} provides useful information on the effectiveness of the reactive power support from each generator. This chapter discusses how to apply the results to the procurement of reactive power support services. There are two potential applications. One is to use Q_{min} as weighting factors in system support cost allocation. The other possible application is to develop a fair compensation scheme to reward generators that do provide reactive power support services.

6.1 Active Power Output Based Allocation Scheme

In some power markets, such as in Alberta, the transmission cost for generators is simply allocated among generators according to their active power output levels. This type of cost allocation can be represented by the following equation:

$$\rho_i = \frac{P_i}{\sum_{j=1}^{k} P_j} C_{trans.gen} \tag{6.1}$$

where ρ_i is the charging rate for the i^{th} generator (in dollars), P_i is its active power output,

k is the total number of generators, ΣP_j is the total active power output from all generators. $C_{trans.gen}$ is the total system support cost to be paid by generators, which is also in dollars. Using this rule to allocate the system support cost among the generators of the BC Hydro system, the following payment scheme can be obtained, as shown in Figure 6.1.

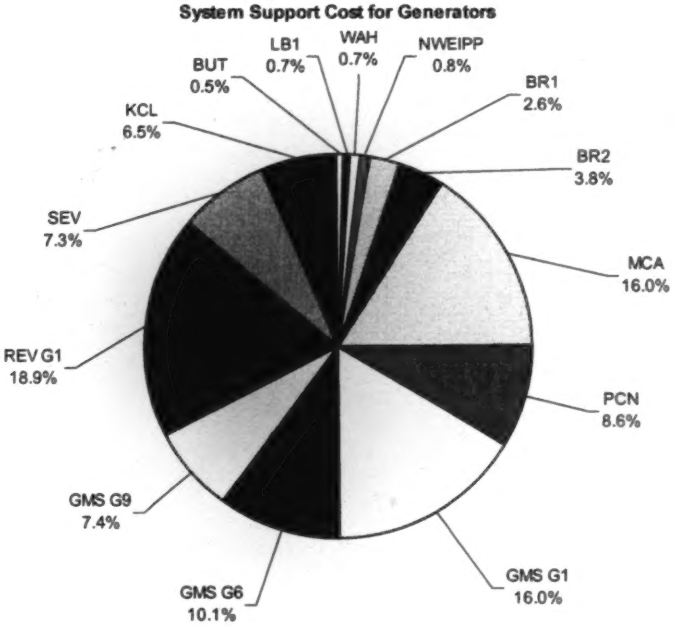


Figure 6.1: Allocation of system support cost according to active power output level

According to this scheme, a large generator or a plant with more active power output needs to pay more than the smaller generators, no matter where the locations of the generators are. Therefore, this scheme provides unfair advantage for the distant generators and treats those important generators that are close to load center unfavorably. Furthermore, it is not reasonable for generators at different locations to have an equal share of the cost when they output the same amount of active power (such as GMS G1 and MCA in Figure 6.1). They are not at the same location and they use the network differently. These disadvantages demonstrate the need for a more fair payment scheme for generators.

6.2 Q_{min} Based Allocation Scheme

The minimum reactive power Q_{min} of a generator is a measure of the generator's need for system support. Therefore, when the system support cost is shared among generators, it should be shared according to the Q_{min} value of each generator. This is especially true for the cost associated with reactive power support. One of the possible ways to take into account the relative importance of a generator's reactive support is to introduce a weighting factor into the cost allocation equation. Since the minimum reactive power requirement (Q_{min}) can effectively measure the generators' need for system support, it is suitable to use Q_{min} as the weighting factor, shown as follows:

$$\begin{aligned} \rho_i &= \frac{W_i P_i}{\sum_{j=1}^{k} W_i P_j} C_{trans.gen} \\ W_i &= Q_{\min.i} \end{aligned} \tag{6.2}$$

where ρ_i is the weighted charging rate of the i^{th} generator (in dollars). W_i, is the weighting factor of the i^{th} generator, which is equal to its Q_{min} value for one MW active power output, i.e., Q_{min}/MW. It is common that the total system support cost is the sum of many different types of cost items. Some cost items may not be related to reactive power support services and should not be allocated according to Q_{min} values. In this case, the $C_{trans.gen}$ in the above equation can be defined as the system support cost related to reactive power support only.

Again, the BC Hydro system is used as an example to illustrate the new allocation scheme. The Q_{min}/MW values of the key generators are obtained using the method described in Chapter 5 and are listed in Table 6.1. Table 6.1 categorizes the key generators according to their types. For example, plant GMS has ten generators. Among the ten generators, five are of the same type as GMS G1, which has active power output of 263MW. Since these five generators have the same Q_{min}/MW value, they belong to one category. The total active power output of this category is then 1305MW.

Table 6.1: Q_{min}/MW values of key generators in the BC Hydro system

Generator Type	**No. of Generators**	***P* output (MW)**	**Q_{min} per MW (Weighting factor W^0) (MVar/MW)**	**Weighting factor W'(MVar/MW)**
BUT G5	2	40	-0.4461	0
LB1	1	55	-0.0846	0.3615
WAH	1	60	0.1274	0.5735
NWEIPP	1	67.5	0.2252	0.6713
BR1	4	208	0.5725	1.0186
BR2	4	306	0.5893	1.0353
GMS G1	5	1305	0.7960	1.2421
GMS G6	3	825	0.8137	1.2597
GMS G9	2	600	0.8615	1.3076
PCN	4	700	0.7724	1.2185
MCA	3	1305	0.6652	1.1113
REV	4	1538.4	1.3454	1.7914
KCL	4	528	3.3171	3.7632
SEV	3	594	2.3844	2.8305

Table 6.1 shows that the Q_{min}/MW values of generator BUT and LB1 are negative. The denominator of Equation (6.2) could be theoretically zero or negative. Although this will not occur in reality (otherwise the system will not need reactive support), its theoretical implication must be addressed. One of the solutions is a so-called minimum payment scheme. This scheme is based on the consideration that all generators utilize the

transmission network to realize their active power transaction. Therefore, they all should participate in the payment of transmission cost, regardless of the sign of their Q_{min} values. However, the generators with negative Q_{min} values should pay less than the other remote generators. The solution is to assign minimum payment to the generator with the most negative Q_{min} value. This minimum payment is sometimes designated and imposed by the system operator. In this case, the new weighting factor for the generator with the most negative Q_{min} becomes positive, i.e., it has been shifted up by ΔW. In order to maintain the relative relationship among all the generators, the weighting factors of all the other generators should also be increased by the same amount ΔW. This procedure is illustrated in Figure 6.2. In this figure, Generator 1 is the one with the most negative weighting factor W_1^0. It is shifted by ΔW to achieve the minimum payment. The other generators, as shown in the figure, are shifted by ΔW also.

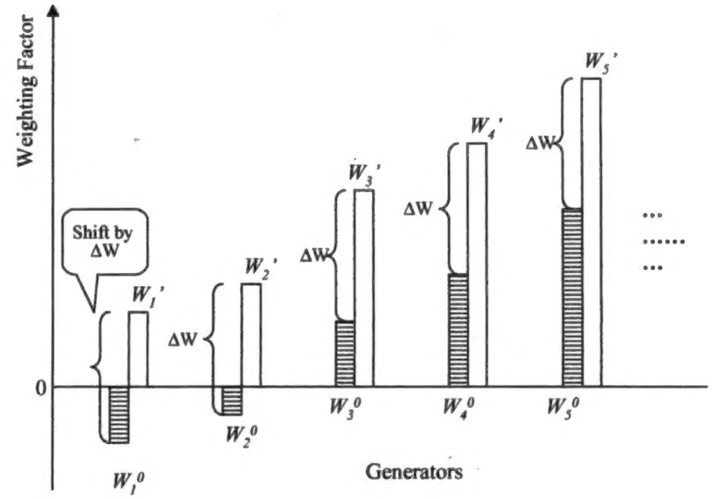


Figure 6.2: Illustration of shifting weighting factors

The shifting amount ΔW, can be determined from the following equations:

$$M = \frac{(W_x^0 + \Delta W)P_x}{\sum_{j=1}^{k}(W_i^0 + \Delta W)P_j} C_{trans.gen} \tag{6.3}$$

where *M* is the required minimum payment. *x* is the generator with the most negative Q_{min} (weighting factor) value. Once ΔW is obtained, the new weighting factor and charging rate for each generator can be determined using the following equations:

$$\begin{gathered} \rho_i' = \frac{W_i' P_i}{\sum_{j=1}^{k} W_i' P_j} C_{trans.gen} \\ W_i' = W_i^0 + \Delta W \end{gathered} \tag{6.4}$$

In the example of the BC Hydro system, BUT is the generator with the most negative Q_{min} value. For illustration, a special case is considered here: zero minimum payment. This effect of zero minimum payment is equivalent to shifting the Q_{min} value of BUT from negative up to zero. The weighting factor of BUT, therefore, becomes zero. The weighting factors of the other generators are shifted up accordingly. The new weighting factors are shown in Table 6.1 as 'weighting factor W''.

Using the obtained new weighting factors and the payment scheme of Equation (6.4), the system support cost is reallocated among the key generators of the BC Hydro system. Figure 6.3 shows the results. It can be seen that this payment scheme is more reasonable than that considering active power output only. For example, compare generator MCA and GMS G1, the two have the same active power output level but at different locations. The new allocation scheme fairly differentiates the two generators and assigns different cost to them according to their minimum reactive power needs. The comparison between the active power output based allocation scheme and the Q_{min} based scheme is given in Figure 6.4.

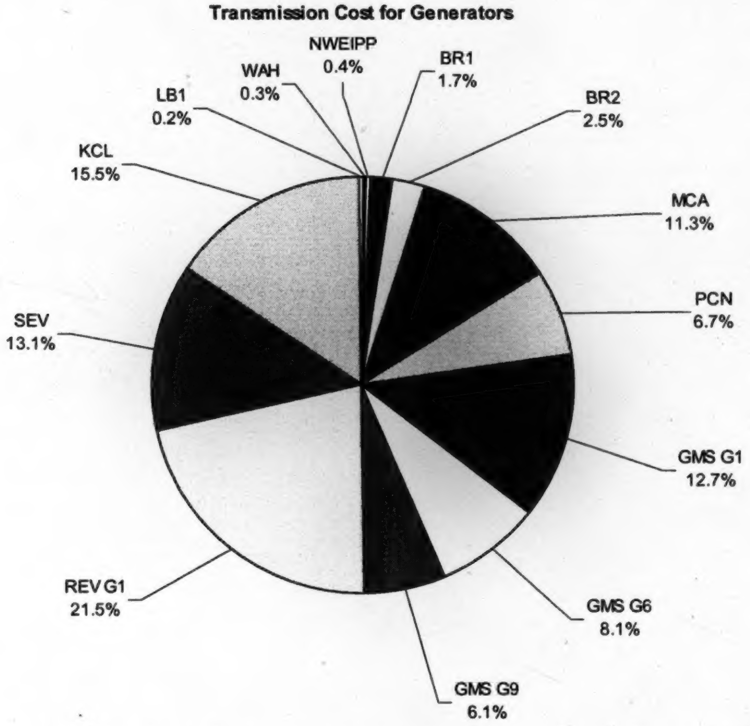


Figure 6.3: Allocation of transmission cost according to weighted *P* output

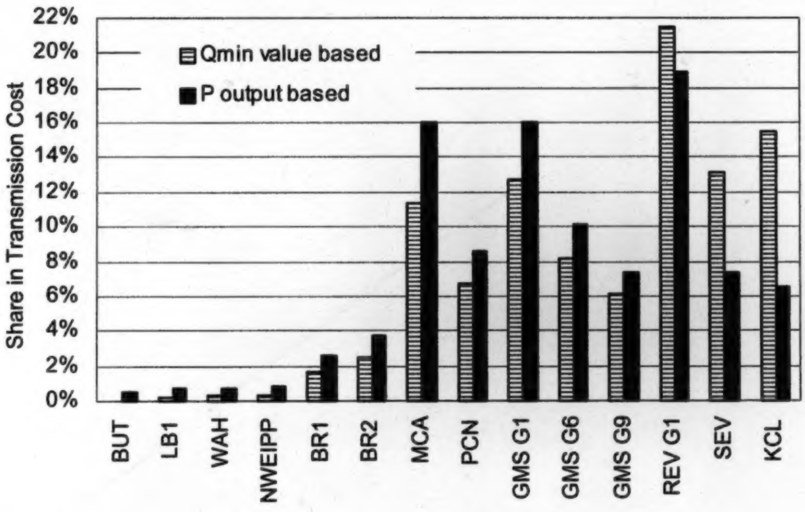


Figure 6.4: Comparison of two schemes for transmission cost allocation

6.3 Compensation for A Generator's Support Service

The cost allocation method described in Section 6.2 is a step forward to create a fair payment scheme for reactive support consumers. It does not, however, provide incentive for generators to produce reactive power. The scheme of Section 6.2 is further improved in this section into a payment as well as compensation scheme for competitive procurement of reactive support services.

At any given time, the reactive power output of a generator can be any value. This value is denoted as Q_{output}. There is also a Q_{min} value associated with the generator at the same time. The system support provided by the generator is actually the difference between the two values as follows,

$$Q_{support} = Q_{output} - Q_{\min} \tag{6.5}$$

It is clear that the $Q_{support}$ can be either positive or negative. Generators with positive $Q_{support}$ shall be compensated while those with negative values shall pay the support received. The proposed method to compensate the generators is as follows:

1) The generators are grouped into two types, the 'provider' type and the 'consumer' type, based on the sign of the $Q_{support}$. The provider type has a positive $Q_{support}$ and the consumer type has a negative value.

2) The provider type shall be compensated. The amount of compensation can be established as follows:

$$\rho_i = \frac{Q_{suppport-i}}{\sum\limits_{i \in provider} Q_{suppport-i}} \rho_{compensation} \tag{6.6}$$

where $\rho_{compensation}$ is the total financial compensation available for the providers.

3) The consumer type shall make payment to the system administrator. The amount of payment can be established as follows:

$$\rho_i = \frac{Q_{suppport-i}}{\sum\limits_{i \in consumer} Q_{suppport-i}} \rho_{cost} \tag{6.7}$$

4) It is important to note that $\rho_{compensation}$ is not equal to and is generally smaller than ρ_{cost}. This is because there are other reactive power sources such as shunt capacitors in the system. The determination of $\rho_{compensation}$ and ρ_{cost} is generally a policy or regulation problem that involves many other considerations.

The BC Hydro system is used to illustrate the application of the above concept. The percentage payment for each consumer type generator is shown in Figure 6.5. It also shows the percent of compensation each provider type generator should be rewarded. It is observed that remote large generators SEV, KCL and MCA share a large percent of payment. Generator BUT should get the most compensation.

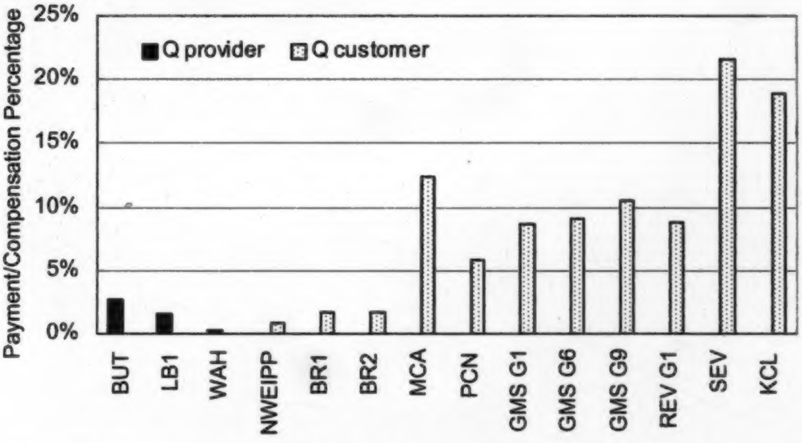


Figure 6.5: Compensation/Payment percentage for key generators

In the above results, the percentages are based on the total cost for reactive support service. It has been assumed, for illustrative purpose, that $\rho_{compensation}$ and ρ_{cost} have the following relationships:

$$\begin{aligned} \rho_{cost} &= C_{trans.gen} \\ \rho_{compensation} &= \frac{\sum_{i \in provider} Q_{support-i}}{\sum_{i \in consumer} Q_{support-i}} C_{trans.gen} \end{aligned} \tag{6.8}$$

The above equation implies that the consumer type generators shall bear all the cost of reactive support service. The provider type generators are entitled for a portion of the cost in the form of financial compensation. This allocation method makes it possible to compare the reward with the penalty for various generators. In the above calculation, all other costs such as transaction cost and administrative cost are excluded from $C_{trans.gen}$.

6.4 Summary

Two possible applications of the Q_{min} results are presented in this chapter. One application is to use Q_{min} results as weighting factors in transmission cost allocation. The second application is to establish a rate structure to compensate those generators that provide reactive support. The following conclusions can be drawn from the results:

- The active power output based cost allocation scheme fails to consider the location differences among the generators. The local generators that are very important for system margin support are at a disadvantaged position since they share the same amount of cost as the remote generators. The Q_{min} index can be used to introduce the location factor into the cost allocation scheme.

- There is a need to differentiate the generators that provide support service from those that consume the service, although all generators may output reactive power. A payment/compensation scheme is presented. This scheme will send incentive signals for generators to improve the management of their reactive power generation. It will foster the development of a truly competitive electricity market.

How to allocate reactive support service cost is a complicated problem that involves not just technical considerations. What presented in this chapter are preliminary ideas and results that highlight the importance of the problem. The Q_{min} method has established some technical foundations to support the development of better schemes in the future.

Chapter 7

Modeling of the Generator Reactive Power Limit [8]

Generators often reach their reactive power limits during the process of system margin calculation. When this happens, a generator is modeled as a constant active and reactive power source. In reality, a generator behaves as a constant voltage source behind an impedance when it reaches the reactive power limit. There is, therefore, a need to clarify which model is more appropriate to represent the generators for the proposed Q_{min} method and for other applications that need to model the generator reactive power limit. The purpose of this chapter is to investigate this issue and to decide which model is more suitable for the proposed application.

7.1 Background

A generator bus is typically modeled as a PV bus in power flow studies and PV/QV analysis. This is due to the fact that the generator can have a sufficient reactive power capability to maintain a constant voltage at its terminal. If a generator's reactive power reserve is depleted, a common practice is to convert the PV bus into a PQ bus. The specified Q for the PQ bus is the generator's Q limit. It implies that the generator outputs constant Q when its Q limit is reached [35]. The assumption of constant Q output is not quite correct, however. The fact is that a generator actually maintains a constant field voltage after reaching its reactive power limit. For regular load flow studies, the errors caused by the PQ bus assumption may be small and there may be little need to refine the modeling method. This situation may not be true, however, for system margin calculations. Experience has shown that a system reaching its PV/QV curve limit is often associated with the depletion of dynamic reactive power support from generators or condensers close to the load centers. Correct modeling of the generator behavior under such conditions can become quite important.

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[8] The work of this chapter has been published: Da Silva, L., **Wang, Y.**, Xu, W. and Da Costa, V.F., "Comparative Studies on Methods of Modeling Generator Var Limit for Power Flow Calculations", *International Journal of Electric Power Components and Systems*, Vol. 29, No. 6, pp. 565-576.

The inadequacy of the PQ modeling method has already been recognized [8, 35-37]. Recent papers also show theoretically and experimentally the importance of considering the generator reactive limit with accuracy [38,39]. In this chapter, a constant field voltage modeling method for the generator reactive power limit, named PE model, is introduced and evaluated. A comparative study between this PE model and the conventional PQ model is performed to assess the characteristics and performance of the two models with respect to power flow computation and system margin assessment. Based on the results, advantages and disadvantages of the two modeling approaches are discussed. The appropriate approach for representing a generator's reactive power limit for this research is identified.

7.2 Modeling of the Generator Reactive Power Limit

The basic power flow model for a generator is shown in Figure 7.1, where E, V_t, and X_s represent the field voltage, terminal voltage and synchronous reactance, respectively. The relationship between the generator's reactive power Q, field voltage E and field current i_f is described in the following equations [8]:

$$\begin{aligned} Q &= \frac{X_{ad}}{X_S} V_t i_f \cos\delta - \frac{V_t^2}{X_S} \\ E &= X_{ad} i_f \end{aligned} \tag{7.1}$$

where δ, X_{ad} represent the generator's rotor angle and d-axis air-gap linkage reactance, respectively.

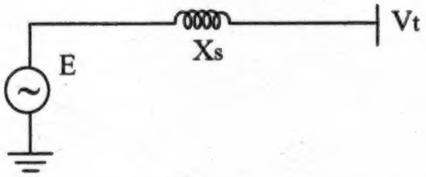
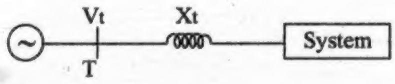


Figure 7.1: Generator equivalent circuit

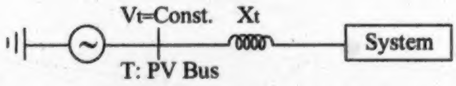
Under normal operating conditions, when the generator has sufficient reactive power reserve, its terminal voltage V_t is maintained nearly constant by adjusting its field current. When V_t decreases as a result of load increases, the field current is increased to produce more reactive power, which helps to keep a constant V_t. When the reactive power reserve of the generator is depleted, however, the terminal voltage can no longer remain constant. The over-excitation limiter, whose function is to protect the generator exciter from overheating, automatically limits the generator field current. The generator, hence, reaches its reactive power limit, and loses the capability of voltage control.

According to Equation (7.1), the field voltage E is in proportion to the field current. Therefore, when a generator is operating on the field current limit, it is the internal voltage E that remains constant instead of the terminal voltage V_t. This internal voltage represents the excitation voltage due to the field current. The point of constant voltage, therefore, is moved to the point behind the synchronous reactance X_s. This model is called the PE model.

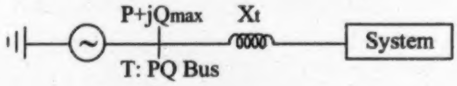
Figure 7.2 illustrates how the PQ and PE modeling methods can be considered for power-flow calculation. A typical scheme of connecting a generator or synchronous condenser to a system is shown in Figure 7.2(a), where X_t represents the equivalent reactance between the generator terminal bus T and the system. Before the generator reaches its reactive power limit, the common practice is to represent bus T as a PV bus, as shown in Figure 7.2(b). In this case the generator's reactive output is adjusted to maintain a constant voltage at the terminal bus T. If the reactive power output reaches its maximum limit, typically Q_{max}, the PQ modeling method converts the PV bus into a PQ bus as shown in Figure 7.2(c). The PQ bus injects a constant reactive power, Q_{max}, into the system.



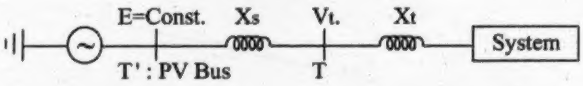
(a). Sample system



(b). Before reaching reactive power limit



(c). PQ model



(d). PE model

Figure 7.2: Generator reactive power limit modeling

Figure 7.2(d) shows the equivalent circuit for the PE modeling method. The specified *P* of the bus is still the generator's active power output and the specified *V* is the maximum field voltage allowed for long-term operation. As it can be seen from the model, the original PV bus now 'retreats' to the bus T'. Its effect is equivalent to an increase of the electrical distance from the generator to the system.

The PE modeling method shown in Figure 7.2(d) can be easily implemented without developing a new load flow formulation. When a generator reaches its reactive power limit, its field voltage is first computed. X_t is then increased to (X_t+X_s) and the specified voltage for the PV bus now becomes the field voltage calculated earlier. With this procedure, the system size remains the same and a conventional load flow program can be used to test the two modeling methods.

7.3 Impact of Modeling Methods on Load Flow Results

According to the analysis of the last section, the PE model seems to be more accurate. It is also easier to implement. The impacts of the two modeling methods on the load flow results are analyzed using a simplified BC Hydro 500kV system [40]. Out of nine equivalent generators just four reach their reactive power limits as the system loading level is increased. In order to compare both solutions for voltage magnitudes and angles, four scenarios are prepared, in which one of the generators is allowed to reach its reactive power limit each time, respectively. For each scenario, the same loading condition is solved by using both PQ and PE models.

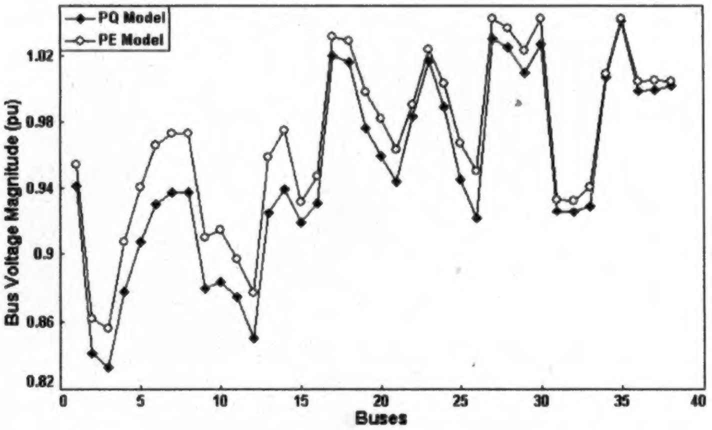
Table 7.1 summarizes the test results obtained for all scenarios. The differences between the two solutions for voltage magnitudes and angles are described as $\Delta V = V_{PE} - V_{PQ}$ and $\Delta\theta = \theta_{PE} - \theta_{PQ}$. Five buses, with the largest differences, are shown in the table. It also shows the average difference between the PQ and PE model, considering all buses in the system.

Table 7.1: Comparison of power flow results

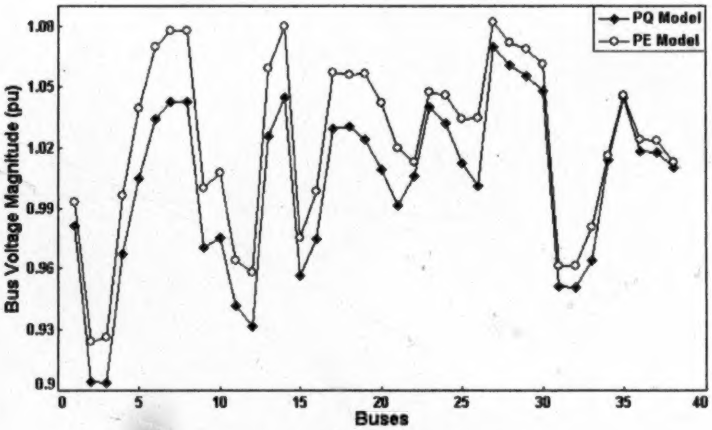
Bus	Generator 1		Generator 2		Generator 3		Generator 4	
	ΔV (% pu)	$\Delta\theta$ (deg)	ΔV (% pu)	$\Delta\theta$ (deg)	ΔV (% pu)	$\Delta\theta$ (deg)	ΔV (% pu)	$\Delta\theta$ (deg)
1	1.18	0.67	1.49	0.98	3.54	2.90	3.56	2.21
2	1.13	0.67	1.43	0.97	3.53	2.90	3.56	2.20
3	1.12	0.67	1.43	0.81	3.52	2.64	3.55	2.02
4	1.08	0.67	1.42	0.81	3.50	2.61	3.53	2.01
5	1.08	0.64	1.40	0.80	3.37	2.54	3.42	1.94
Avg.	0.57	0.24	0.64	0.31	1.83	1.05	2.06	0.97

It can be concluded from Table 7.1 that the biggest difference for voltage magnitude is below 5% and for voltage angles less than 3 degrees. Although the PE model can provide the most accurate load flow results, the PQ model does not produce large errors, since the average difference (Avg.) is below 5%. This means that the traditional PQ model, although it is an approximation, still has sufficient precision in terms of power flow results.

As a further illustration of the two models' impacts on load flow results, voltage magnitudes for all load buses are plotted in Figure 7.3. Figure 7.3(a) shows the case where Generators 3 hits the reactive power limit and Figure 7.3(b) pictures the case for Generator 4. With the PE model, the voltage magnitudes for all buses are a little higher than those of the PQ model. But the differences are not significant, as discussed before.



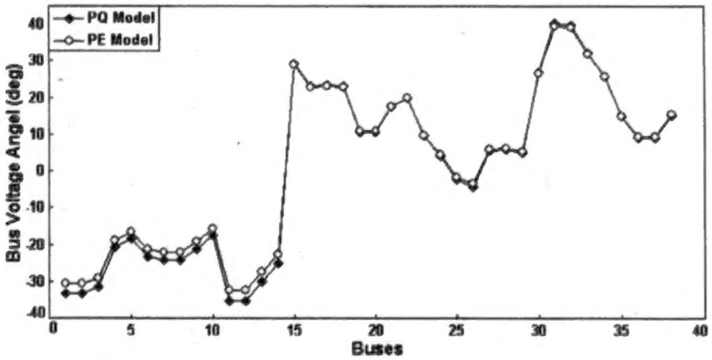
(a). Generator 3 hits var limit



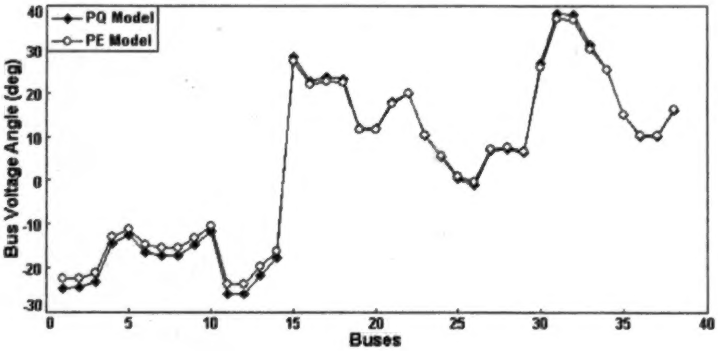
(b). Generator 4 hits var limit

Figure 7.3: Voltage magnitude comparison

Figure 7.4 (a) and (b) also show comparison results for voltage angles. No considerable differences can be observed from the figures. Similar results are obtained when Generator 1 and 2 are allowed to hit the var limits.



(a). Generator 3 hits var limit



(b). Generator 4 hits var limit

Figure 7.4: Voltage angle comparison

7.4 Impact of Modeling Methods on Voltage Stability Margin

The simple system of Figure 7.5 is first used to assess the impact of the two modeling methods on voltage stability margin. The purpose of using such a simple system is to make the results easier to understand. This system has a distant generator (bus 1) to serve as a reference (slack) bus. It supplies little active and reactive power to the system. The principal generator is the one connected to bus 3. This generator is modeled using two methods, the conventional PQ model and the constant field voltage PE model, when its reactive power limit is reached.

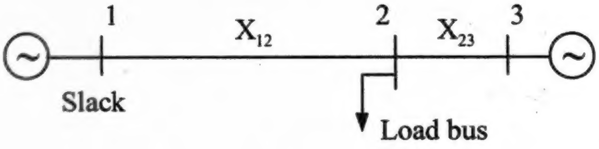


Figure 7.5: Sample test system

Figure 7.6 shows the PV curves for different Q_{max} values. It can be noted that the conventional PQ model is an acceptable approximation of the PE model, since it only gives slightly different margins.

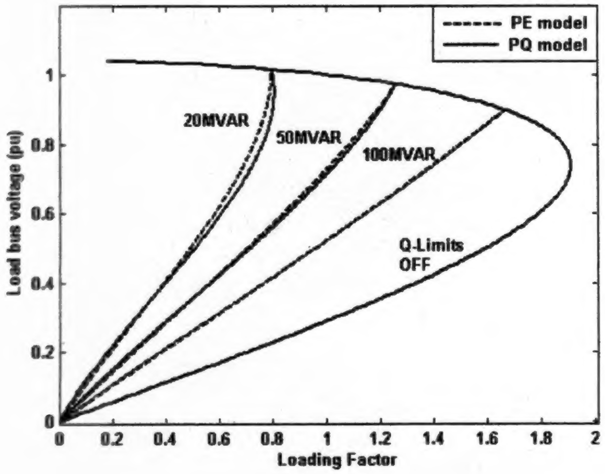


Figure 7.6: Impact on the PV curve with different reactive limits

A more detailed study is performed on the BC Hydro 500kV system. This is a highly loaded and compensated system with clear characteristics of voltage instability problems. Figure 7.7 shows the PV curves of this system by allowing all generators to hit their reactive power limits. The reactance of each generator is assumed to be equal on the basis of generator ratings. In this case, the PQ model exhibits a stability margin around 1% less than that of the PE model.

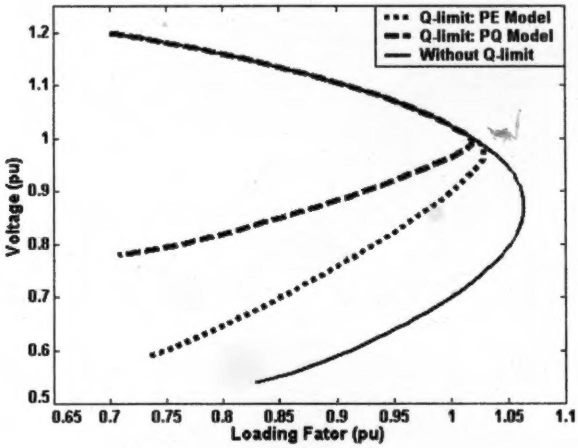


Figure 7.7: PV curves of the BC Hydro 500kV system

Table 7.2 summarizes the impact of the two models on voltage stability margins for all the considered scenarios, which are the same as in Section 7.3. The fifth case considers all of the four generators hitting their var limits. For each case the maximum system load-ability is obtained by using both PQ and PE models to represent the reactive limits. The difference of margin in percentage between the two models is computed. Based on the results, it is concluded that the conventional PQ model seems to be reasonable in terms of PV margin results.

According to the information provided in Table 7.2, the PE modeling method provides a larger margin than that of PQ modeling method in this case. It should be noted that the voltage stability margin obtained from PE modeling method will change with respect to different generator synchronous reactance. A smaller synchronous reactance will produce a larger margin, while a larger synchronous reactance will decrease the margin.

Table 7.2: Voltage stability margin comparison

Scenarios	Maximum load-ability (pu)		Percentage Difference
	PE Model	PQ Model	
Generator 1	1.0408	1.0395	0.1%
Generator 2	1.0605	1.0556	0.5%
Generator 3	1.0605	1.0589	0.2%
Generator 4	1.0357	1.0202	1.5%
All generators	1.0293	1.019	1%

Figure 7.8 shows the sensitivity of the PV curve with respect to the synchronous reactance for the simple system of Figure 7.5. It can be observed that the PQ model can be more optimistic or conservative depending on the value of the synchronous reactance. These results are confirmed on a nine-bus system [7] and an IEEE 39 bus (New England) test system, as shown in Figure 7.9. The figures illustrate that the margin is slightly decreased in the nine-bus system and slightly increased in the New England system.

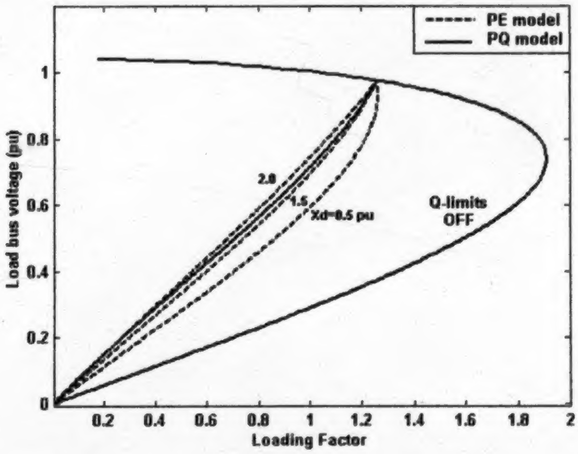
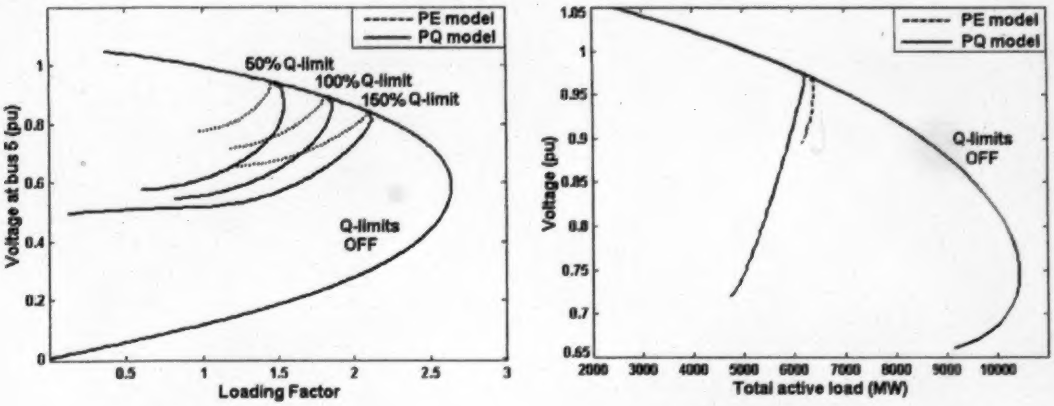


Figure 7.8: Sensitivity of the PV curve to different synchronous reactance values



(a). Nine-bus system (b). New England system

Figure 7.9: Optimistic and conservative PV curves

7.5 Impact of Modeling Methods on Convergence Characteristics

Superficially, the PE model seems to provide better convergence characteristics than the conventional PQ model, since the PE model maintains the same number of PV buses in the PV curve tracing process, while the PQ model leads to a reduced number of PV buses. It is commonly known that a system with more PV buses tends to exhibit improved convergence characteristics. After careful investigation, however, the PE model was found to be poorer in convergence, and the conventional power flow method, in most cases, was found to be unable to solve the load flow problem with the field voltage included. The causes are identified as follows:

- The PE model can result in a very sharp turning of the PV curve (e.g. Figure 7.6). Once the reactive power limit is reached, the operating point changes to the one below the nose point. Conventional load flow methods are known to have difficulties to converge for the points below the nose point.

- The field voltages (from 2 to 4 per unit) have values much higher than those of the bus voltages. The abrupt PV bus voltage changes brought by generators reaching reactive power limits also cause difficulties in load flow convergence.

To illustrate the differences of the two modeling methods in convergence features, a continuation power flow is used [41-43]. Since the same continuation method is applied for the two models, the results can be compared. Figure 7.10 shows the number of iterations for each PV curve point of the simplified BC Hydro system. Each disturbance in the iteration number means that a generator reaches its reactive power limit. The figure shows that the PE model needs one or two more iterations than the conventional PQ model. It is also found that larger synchronous reactance generally result in poorer convergence characteristics.

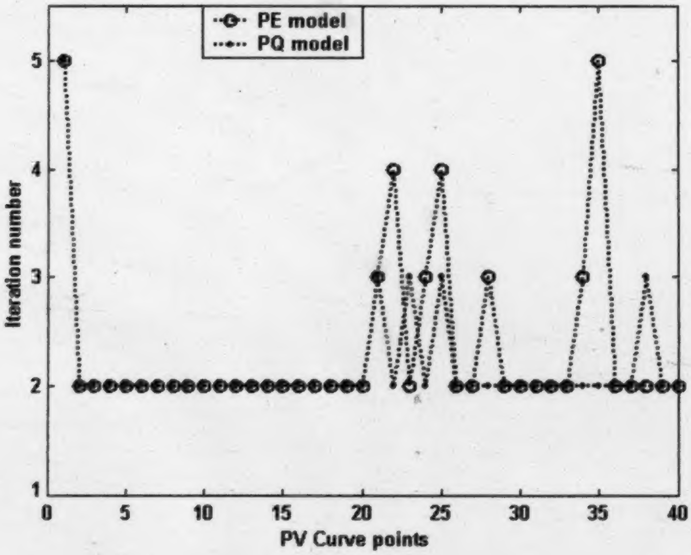


Figure 7.10: Comparison of convergence characteristics

7.6 Summary

This chapter conducts comprehensive investigations on the impacts of using the PE modeling method and the PQ modeling method to represent a generator's reactive power limit. Theoretically speaking, the traditional PQ modeling method, is an approximate method. It is the PE modeling method that accurately represents the generator behavior when the reactive power limit is reached. The PE model is also easier to implement than the conventional PQ model, since it does not require changing the structure of the load flow Jacobian matrix. The differences of the two modeling methods are assessed from three perspectives of power flow calculation and voltage stability assessment. The main conclusions of this work are summarized as follows:

- Although the PE model is the most accurate model for representing generators reaching their reactive power limits in steady state computations, it provides load flow results, and voltage stability margins just slightly different from the ones obtained by the conventional PQ model. These results are supportive to the use of the conventional PQ model. Numerical measurements of the errors associated with the PQ model are presented, which are very small.
- Due to the inclusion of the field voltage, which is usually much higher than the normal bus voltage, and its dependence on the generator's synchronous reactance, the convergence of the PE model is found to be poorer compared with the convergence characteristics of the conventional PQ model. This represents another advantage of the PQ model.

It is possible that the two modeling methods may have different impacts on calculation results when other aspects of power system analysis are of interest. However, they are not of concern in this thesis research. Based on the technical need of this research and the discussion of the pros and cons of the PQ model in comparison with the PE model, the PQ model is recommended for power system margin assessment.

Chapter 8

Analysis of Ill-Conditioned Power Flow Problems [9]

In the early stage of this thesis research, the ill-conditioned power flow problem was investigated since it was encountered frequently when calculating system margins. The divergence of a power flow problem using the conventional power flow method is often associated with power flow ill-conditioning. Typical approaches for this problem are to develop enhanced solution algorithms. However, these works seldom investigate the real cause of the divergence. It is known that a genuine ill-conditioned problem is caused by the presence of a large condition number in the matrix [44], the matrix of concern in the power flow problem is the Jacobian matrix. Since a large condition number is associated with small singular values or eigenvalues of a matrix and the voltage collapse is also related to small eigenvalues, it is therefore natural to postulate that an ill-conditioned power flow problem may be just a voltage collapse problem. The objective of this research is to clarify the relationship between power flow ill-conditioning and voltage instability. The results will be useful to guide the treatment of ill-conditioned power flow cases.

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[9] The work of this chapter has been published: **Wang, Y.**, Da Silva, L.C.P., Xu, W. and Zhang, Y., "Analysis of Ill-Conditioned Power Flow Problems Using Voltage Stability Methodology", *IEE Proceedings on Generation, Transmission and Distribution*, Vol. 148, No. 5, pp. 384 –390.

8.1 Background Information

Many efforts have been made, even in recent years, to find efficient methods for solving the power flow of the so-called ill-conditioned power systems [45-51]. Typical research strategies in these works can be summarized as follows: 1) the divergence of a power flow problem using the conventional Newton method is observed; 2) the system is then labelled as ill-conditioned; and 3) new solution algorithms are developed to solve the problem. These works seldom investigate the real cause of the divergence. There is no explanation as to whether the divergence is just a computational problem or a physical limitation of the power system involved.

There is a strict definition for ill-conditioned problems in mathematics [44]. A matrix is considered to be ill-conditioned if it has a sufficiently large condition number. For power flow problems, the matrix of concern is the Jacobian matrix. A large condition number is generally associated with small singular values or eigenvalues of a matrix, and the voltage collapse is also related to small eigenvalues[52]. These lead to the postulation that an ill-conditioned power flow problem could actually be a voltage collapse problem. The objective of this work is to determine the validity of this postulation.

This research study, therefore, adopts a completely different approach to assess the ill-conditioned or divergent power flow problems. It applies the PV curve technique to determine whether solutions actually exist for some well-publicized ill-conditioned power flow problems [45-49]. The distances between the solutions and the PV curve nose points are also determined. The behaviours of singular values and condition numbers are monitored along the PV-curve trajectories. The findings of this work have confirmed that power flow ill-conditioning only occurs at the voltage collapse point. As a result, the well-known voltage stability assessment techniques such as the PV curve method are sufficient for solving the problem. This conclusion is also supported by rigorous mathematical analysis.

8.2 Defining Ill-Conditioned Power Flow

Mathematical theory defines ill-conditioned matrices as those that have sufficiently large condition numbers [44]. For power system analysis, the matrix of interest is the power flow Jacobian matrix $\boldsymbol{J}$. According to [44], the condition number of $\boldsymbol{J}$ is defined as

$$Cond(\boldsymbol{J}) = \|\boldsymbol{J}\| \bullet \|\boldsymbol{J}^{-1}\| \tag{8.1}$$

where ||*|| represents matrix norm. If 2-norm is used, the condition number can be calculated using the following equation [44]

$$Cond(\boldsymbol{J}) = \sigma_{\max}(\boldsymbol{J}) / \sigma_{\min}(\boldsymbol{J}) \tag{8.2}$$

where σ_{max} and σ_{min} represent the maximum and minimum singular values of the Jacobian matrix $\boldsymbol{J}$ respectively.

Equation (8.2) suggests that there are two possible causes leading to a large condition number. One is a very large σ_{max} and the other is a very small σ_{min}. Since the Jacobian matrix entries are comparable to those of the network admittance matrix, it is reasonable to assume that σ_{max} cannot be very large. The probability for σ_{max} to cause a large condition number is therefore small. On the other hand, extensive voltage stability research results show that σ_{min} (or one of the Jacobian eigenvalues) can be very small or zero at the voltage collapse point [52]. One can, therefore, postulate that the only cause for a large condition number to occur would be σ_{min} going to zero at the PV curve nose point. Thus the ill-conditioned power flow problem could be a mere voltage stability problem, caused by operating the system at its maximum loading level. To verify this postulation, the PV curve method is used to assess five widely publicized ill-conditioned power flow systems [45-49].

When a system can not be solved with conventional power flow programs, no clues are available to determine the cause of divergence and the condition numbers since there are no solved results. The PV curve technique, however, can reveal a lot of useful information. PV curve can always be computed for a system by scaling down the system load and generation, and by using a continuation method [41]. As shown in Figure 8.1, there are three possible scenarios for power flow divergence:

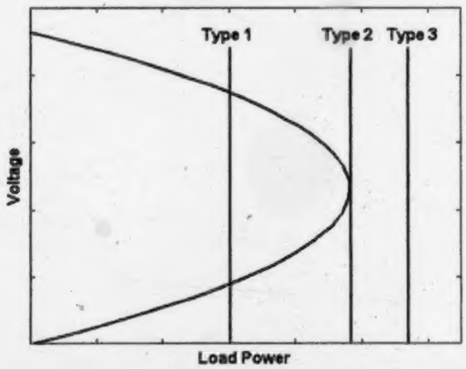


Figure 8.1: Possible load flow solutions as seen from the PV curve

Type 1: A solution point exists and it is not close to the PV curve nose point. The Newton method should have no problem to solve such a case in theory. Whether the problem is ill-conditioned or not can be easily checked using the condition number at the solution point.

Type 2: There is a possible solution point but it is very close to the PV curve nose point. The Newton method is known to have difficulties to solve such a case. If the condition number is indeed very large for such a case, the problem can be classified as ill-conditioned.

Type 3: A solution point does not exist. In other words, the loading level of the case has gone beyond the system capability. Such cases have nothing to do with the ill-conditioning of the Jacobian matrices.

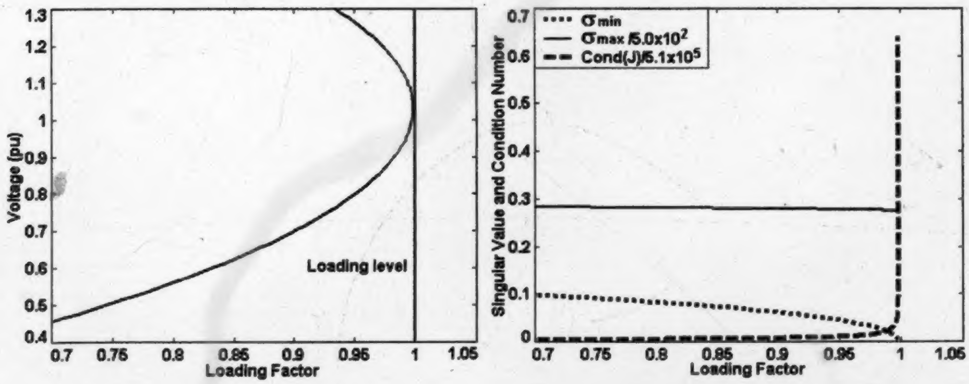
In addition to providing the 'locations' of solution points, the PV curve method allows one to assess the behaviour of the Jacobian matrix condition number and singular values around the solution points. Such information is useful to reveal the causes of power flow divergence and to determine if a case is indeed ill-conditioned. The next section presents detailed studies on five cases using the PV curve methodology.

8.3 Case Studies

Case studies are performed on five classical 'ill-conditioned' power flow cases cited in references [45- 49]. A conventional Newton method is first used to solve the cases. If a case is divergent, its PV curve is then computed, and the singular values and condition number of the Jacobian matrix are monitored. The PV curve is obtained by increasing all active and reactive loads of the system proportionally in steps. The generation is also scaled up accordingly. The voltage of a randomly selected load bus is then plotted with respect to the load level increase. In this study, the load level (x-axis) is expressed in the form of 'loading factors'. A loading factor is the ratio of the actual system load level to the base case level. A loading factor of 1 therefore represents the original or base case. A loading factor greater than 1 indicates an increased system load level. Detailed results for each case are presented as follows.

8.3.1 11-Bus System

This case is reported as ill-conditioned in [47], since it was found divergent with the Newton method. The authors attributed the divergence to low *X*/*R* ratios and negative line reactance. A modified Newton method was thus proposed. The test results obtained in this chapter confirm that the case cannot converge using the conventional Newton method. However, the PV curve results, shown in Figure 8.2(a), reveal that the base case loading level is 99.8% of the maximum load-ability of the system. It means that the case is very close to having no feasible solution. It is thus classified as Type 2. Figure 8.2(b) shows the behaviour of the condition number, and the maximum/minimum singular values while the system loading is increased. It is clear that the maximum singular value remains at a finite value for the entire curve, while the minimum singular value goes to zero at the nose point. It is only near the nose point that the condition number becomes very large. Therefore, the case is a genuine ill-conditioned case.



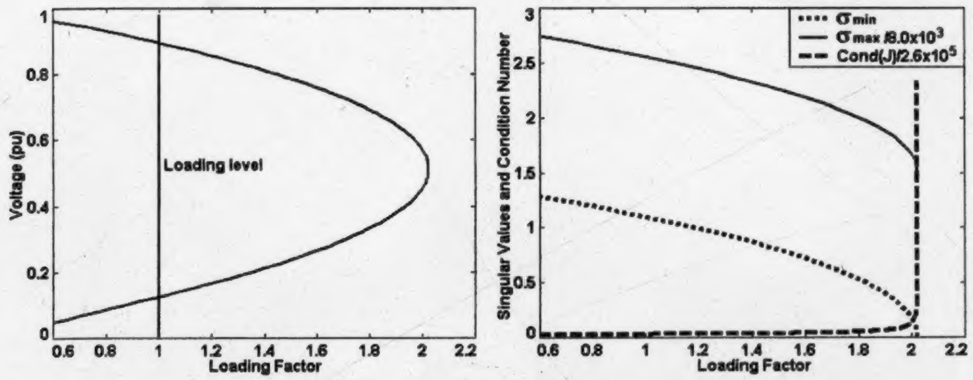
(a). PV curve

(b). Singular values and condition number

Figure 8.2: Characteristics of the 11-Bus System

8.3.2 30-Bus System

This case is a radial distribution system. Reference [45] considers the case as ill-conditioned since it does not converge after 50 iterations with the Newton method. However, the results of this chapter show that the case is convergent after 3 iterations with the Newton method. The maximum system loadability is 2.024 times of the base case loading level, indicating a large margin from the base case to the nose point. Figure 8.3 shows the PV curve and condition number results. No large condition number is observed at the solution point. This case is therefore not ill-conditioned and can be classified as Type 1. The divergence problems found in [45] might be related to other numerical problems such as an inadequate starting point for iterations. Figure 8.3(b) also confirms that a large condition number only exists near the voltage collapse point.

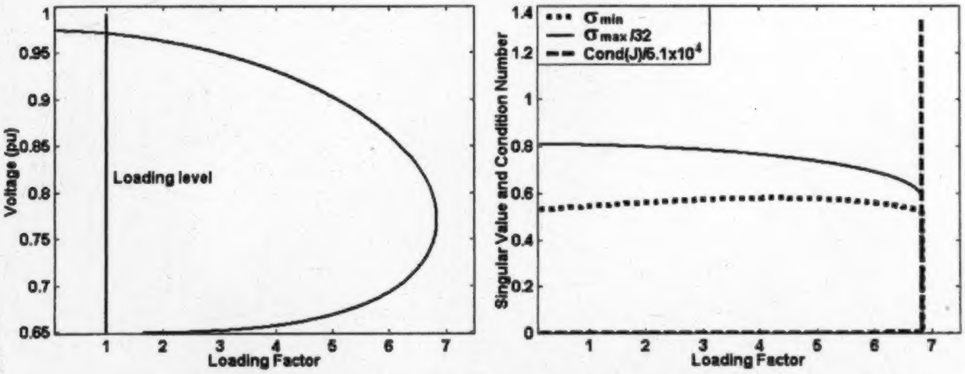


(a). PV curve (b). Singular values and condition number

Figure 8.3: Characteristics of the 30-Bus System

8.3.3 13-Bus System

This system consists of three transformers, three generators and three condensers. It is described in [47] that the Newton method failed to converge because of the two series capacitors and the position of the slack generator. After analysing this system, errors were found in the input data of the reference. The authors used off-nominal tap settings of 0.05, 0.1, 0.1 for the three transformers. When the tap settings are changed to 1.05, 1.1, 1.1, the Newton power flow converged after four iterations. The incorrect tap settings are found as the only cause of divergence for this system. The PV curve and condition number for this system are shown in Figure 8.4. The nose point occurs at 6.8 times of the base case loading level. The condition number for the solution point is 495.3. This is a Type 1 case and it is not ill-conditioned.



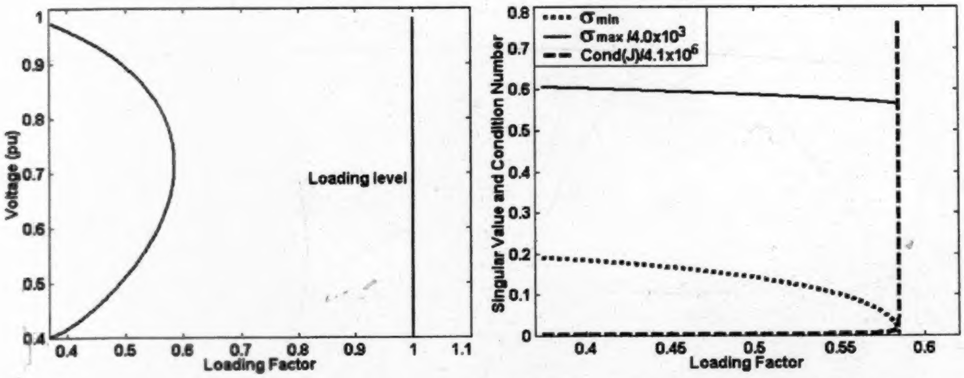
(a). PV curve (b). Singular values and condition number

Figure 8.4: Characteristics of the 13-Bus System

8.3.4 43-Bus System

This case represents a typical distribution system [45-47, 49]. In [45] it is reported as 'specially ill-conditioned' and divergent after 50 iterations using the Newton method.

It is confirmed in this chapter that the case is divergent. However, the PV curve results shown in Figure 8.5 reveal that the nose point occurs at a loading factor of 0.58. In other words, the base case loading level is about 42% higher than the maximum load-ability of the system. There is no feasible solution to the problem. This case is not an ill-conditioned problem and should be classified as Type 3. Results shown in Figure 8.5(b) also confirm the postulation that a very large condition number occurs only at the nose point.



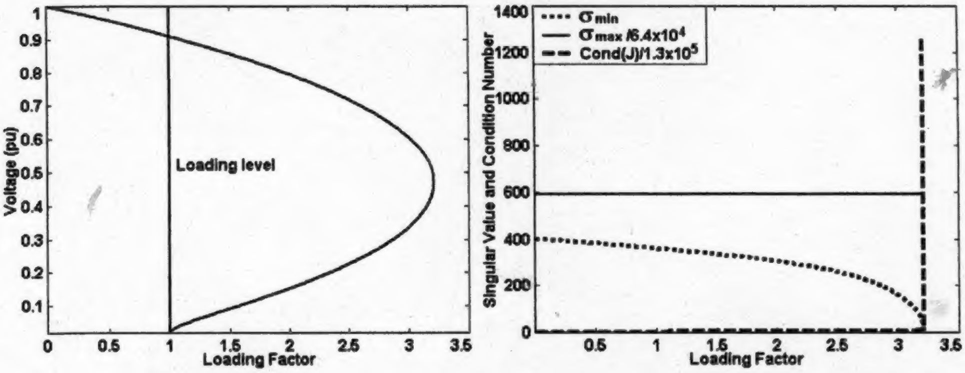
(a). PV curve

(b). Singular values and condition number

Figure 8.5: Characteristics of the 43-Bus System

8.3.5 69-Bus System

This system is a portion of a distribution network. Reference [45] reported it as ill-conditioned. However, the case was found in this research work to reach convergence after four iterations with the Newton method. The PV curve in Figure 8.6(a) shows that the nose point has a loading of 3.21 times of the base case loading. Shown in Figure 8.6(b), the condition number is not large at the solution point (loading factor=1) and remains almost constant till loading factor of 3. However, it goes to infinity when approaching the nose point.



(a). PV curve (b). Singular values and condition number

Figure 8.6: Characteristics of the 69-Bus System

8.3.6 Summary

Table 8.1 summarizes the results obtained for all five cases. According to the condition numbers shown in the table, only the 11-bus case is a genuine ill-conditioned case. The case actually operates at the voltage collapse point. The table also shows that the condition numbers are not large when the solution points are far away from the nose points. The divergent problem of other cases reported in the references, is due to problems other than large condition numbers.

Table 8.1: Case studies summary

Test Systems	Iteration Numbers	Type	Max. loading factor	Condition Number	
				near solution point	Near nose point
11-bus	Divergent	2	0.9981	N/A	$3.3\text{x}10^5$
30-bus	3	1	2.0244	$1.6\text{x}10^4$	$5.9\text{x}10^5$
13-bus	4	1	6.8402	495	$6.9\text{x}10^4$
43-bus	Divergent	3	0.5846	N/A	$3.1\text{x}10^6$
69-bus	4	1	3.2116	$5.8\ \text{x}10^5$	$1.6\text{x}10^8$

8.4 Mathematical Analysis

All case study results have shown that the ill-conditioning of a power flow Jacobian matrix only occurs at the PV curve nose point. In other words, ill-conditioned power flow problem is essentially a voltage instability problem. In this section, mathematical analysis is performed to prove that this is a general conclusion. It is shown analytically that a very large condition number occurs only at the voltage collapse point. A voltage collapse point is defined as the PV curve nose point, where Jacobian matrix has a zero eigenvalue λ.

According to the voltage stability theory [52], σ_{min} or λ_{min} becomes very small or zero only near or at the PV curve nose point. If it can be shown that σ_{max} remains a finite number along the PV curve trajectory, one can easily conclude that the only condition leading to a very large condition number will be σ_{min} approaching zero. The mathematical proof on the upper bound of σ_{max} is given below.

Singular value of the Jacobian matrix σ_i (***J***) can be obtained by the following equation [44]:

$$V^T(J^T J)V = diag(\sigma_{\max}^2,\cdots,\sigma_i^2,\cdots,\sigma_{\min}^2) \tag{8.3}$$

where ***V*** is an orthogonal matrix. If $A=J^T J$ is defined, there exists

$$V^T A V = diag(\lambda_{\max},\cdots,\lambda_i,\cdots,\lambda_{\min}) \tag{8.4}$$

where λ_i , λ_{max} , λ_{min} are the i_{th} , maximum and minimum eigenvalues of matrix ***A***, respectively. Combining Equation (8.3) and (8.4), it yields

$$\lambda_{\max}(\boldsymbol{A}) = \lambda_{\max}(\boldsymbol{J}^T\boldsymbol{J}) = \sigma_{\max}^2(\boldsymbol{J}) \tag{8.5}$$

Noted that $\lambda_i(\boldsymbol{J}^T\boldsymbol{J}) \geq 0$. According to the definition, the eigenvalue of a matrix $\boldsymbol{A}$ can be determined as follows:

$$\boldsymbol{A}\boldsymbol{x}_i = \lambda_i \boldsymbol{x}_i \tag{8.6}$$

or

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = \lambda_1 x_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = \lambda_2 x_2 \\ \cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = \lambda_n x_n \end{cases} \tag{8.7}$$

where $\boldsymbol{X}$ represents the right eigenvector of $\boldsymbol{A}$. Let $|x_m|$=max$|x_j|$, the corresponding m^{th} equation in Equation (8.7) can be expressed as:

$$(\lambda_m - a_{mm})x_m = \sum_{\substack{j=1 \\ j\neq m}}^{n} a_{mj}x_j \tag{8.8}$$

Taking absolute value on both sides of Equation (8.8) yields

$$|\lambda_m - a_{mm}||x_m| = \left|\sum_{\substack{j=1 \\ j\neq m}}^{n} a_{mj}x_j\right| \leq |x_m| \sum_{\substack{j=1 \\ j\neq m}}^{n} |a_{mj}| \tag{8.9}$$

Since $|x_m| \neq 0$, the following condition will hold

$$\left|\lambda_m - a_{mm}\right| \le \sum_{\substack{j=1\\ j\neq m}}^{n} \left|a_{mj}\right| \tag{8.10}$$

Equation (8.10) can also be expressed as

$$a_{mm} - \sum_{\substack{j=1\\ j\neq m}}^{n} \left|a_{mj}\right| \le \lambda_m \le \sum_{\substack{j=1\\ j\neq m}}^{n} \left|a_{mj}\right| + a_{mm} \tag{8.11}$$

Considering that $a_{mm} \le \left|a_{mm}\right|$, Equation (8.11) can be extended as

$$\lambda_m \le \sum_{\substack{j=1\\ j\neq m}}^{n} \left|a_{mj}\right| + \left|a_{mm}\right| \tag{8.12}$$

As a result, the following upper bound is obtained for λ_m

$$\lambda_m \le \sum_{j=1}^{n} \left|a_{mj}\right| \tag{8.13}$$

The above equation applies to each λ_i, Assuming

$$M = \max\left(\sum_{j=1}^{n} \left|a_{ij}\right| \right) \qquad i = 1,2,\cdots n \tag{8.14}$$

one can conclude that the largest eigenvalue is bounded by the following equation:

$$\lambda_{\max} \le M \tag{8.15}$$

According to Equation (8.5), the above condition can also be stated as

$$\sigma_{\max}^2(J) \le M \tag{8.16}$$

or

$$\sigma_{\max}(J) \le \sqrt{M} \tag{8.17}$$

Since all elements in matrices J and $J^T J$ have finite values, Equation (8.17) has proven that σ_{max} has a limited value comparable to the entries of the J matrix. As a result, the only possibility for the condition number of the J matrix to become very large is at the point when σ_{min} approaches zero, namely, the voltage instability point. This conclusion has also been demonstrated by the numerical results of Section 8.3.

8.5 The Application of Continuation Power Flow

The continuation power flow method [41] has been considered as the most effective method to compute PV curves of a power system. A significant characteristic of the method is that it can determine the PV curve section near and below the nose point. It therefore appears that the method is somewhat immune to the ill-conditioning of the Jacobian matrix near the nose point. This possibility has two implications. Firstly, if the method can indeed alter the condition number of the Jacobian matrix, the continuation power flow could be considered as a fundamental solution for ill-conditioned power flow problems. Secondly, a simple approach for testing power flow ill-conditioning could be developed from this understanding. The purpose of this section is to investigate such possibilities.

The continuation power flow method is based on the following conventional Newton power flow equations

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = J \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \tag{8.18}$$

It first introduces a continuation parameter that is typically the loading factor γ. The power flow equation is transformed to:

$$\begin{aligned} \gamma \boldsymbol{P}^{sp} - \boldsymbol{P}(\boldsymbol{\theta},\boldsymbol{V}) = 0 \\ \gamma \boldsymbol{Q}^{sp} - \boldsymbol{Q}(\boldsymbol{\theta},\boldsymbol{V}) = 0 \end{aligned} \tag{8.19}$$

where $\boldsymbol{P}^{sp}$ and $\boldsymbol{Q}^{sp}$ represent active and reactive net power injection respectively. By treating variable γ as an 'unknown' state variable and a pre-selected PQ bus voltage (bus k) as a 'known' parameter [41], Equation (8.19) can be linearized to get the following power flow equations:

$$\begin{bmatrix} \Delta \boldsymbol{P} \\ \Delta \boldsymbol{Q} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H} & \boldsymbol{N}' & -\boldsymbol{P}^{sp} \\ \boldsymbol{M} & \boldsymbol{L}' & -\boldsymbol{Q}^{sp} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta \boldsymbol{V}' \\ \Delta \gamma \end{bmatrix} = J_V \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta \boldsymbol{V}' \\ \Delta \gamma \end{bmatrix} \tag{8.20}$$

where $\boldsymbol{J}_V$ is the expanded Jacobian matrix. Vector $\Delta \boldsymbol{V}'$ does not contain element ΔV_k, which is replaced by $\Delta\gamma$. Consequently, the elements in column k of matrices $\boldsymbol{N}'$ and $\boldsymbol{L}'$ are derivatives of power with respect to γ.

The extended Jacobian matrix $\boldsymbol{J}_V$ avoids the singularity problem. One can, therefore, expect that the condition number of $\boldsymbol{J}_V$ will be reduced significantly at the nose point. The objective here is to verify this conclusion using the five test cases. Figure 8.7 shows a comparison between the condition numbers of $\boldsymbol{J}$ and $\boldsymbol{J}_V$ for the 43-bus test system. It can be seen that the condition number of $\boldsymbol{J}_V$ remains almost constant near the nose point, while that of the conventional $\boldsymbol{J}$ matrix raises significantly. Figure 8.8 shows the condition numbers of matrices $\boldsymbol{J}_V$ for all test systems. All results have shown that the condition numbers have, indeed, been reduced significantly. Compared with other published methods that solve the ill-conditioning problem, the continuation method deals with the core cause of the problem – large condition number – directly. One could therefore consider it as a direct or 'ultimate' solution for the ill-conditioned power flow problems. In addition, the method can provide power flow results around the solution point. The causes of power flow divergence can be easily identified with the information.

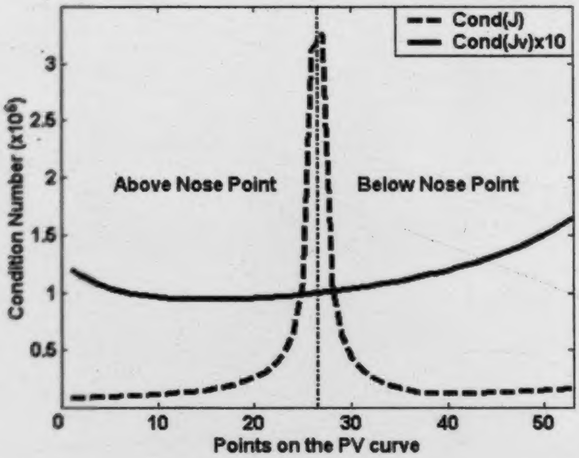


Figure 8.7: Comparison of condition numbers (43-bus system)

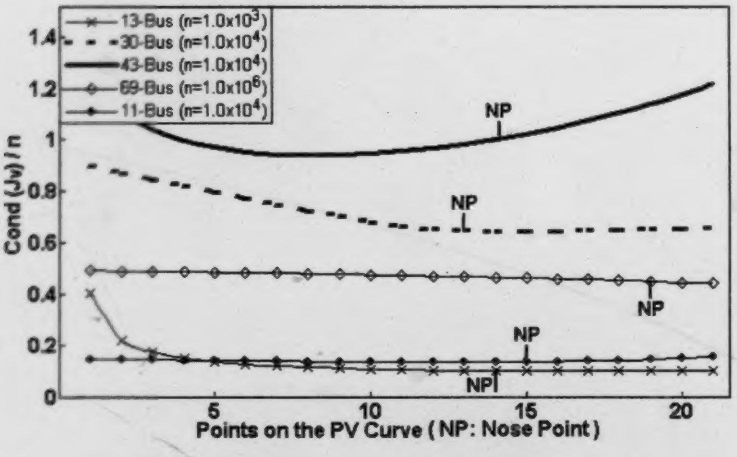


Figure 8.8: Condition number of J_V

QV curve, an easy-to-implement and well-known voltage stability assessment technique, is a special version of the continuation power flow method. The method also has a clear physical meaning since the continuation variable is the voltage of the test bus. Based on the results of this section, the QV curve technique is proposed as a tool to check if a divergent power flow case is a genuine ill-conditioned case. The procedure is as follows:

1). Select a test bus for the divergent power flow case. This could be a bus with the largest load or a bus most remote to the generators of the system.

2). Add a fictitious condenser to the test bus. The voltage setting of the condenser is varied to form a QV curve for the system.

3). Assume that the QV curve is obtained and the condenser output is Q_b at the bottom of the curve. If $Q_b \approx 0$, the case is near the nose-point and is therefore a genuine ill-conditioned case. The 11-Bus system is such an example. If $Q_b >> 0$, the case is over-stressed and there is no feasible solution. The 43-Bus system is such an example. If $Q_b << 0$, the case has a feasible solution and is not ill-conditioned. The power flow divergence could be caused by, for example, excessively high bus voltages. The 30-Bus system is such an example.

4). If the QV curve cannot be obtained or the QV curve bottom cannot be determined, one can still conclude that the case is not an ill-conditioned case. But it cannot be determined if the power flow divergence is due to excessive loading or other factors.

8.6 Discussions

This chapter defines an ill-conditioned power flow problem as the one whose Jacobian matrix has a large condition number. A more fundamental definition of the problem could be that a small change in the system parameters causes a large change in the solution results. This definition is independent of the solution methods. Another related problem is: is it possible that there could exist 'other types' of genuine ill-conditioned power flow problems which are somewhat independent from system loading levels? Possible examples are cases that contain extreme variations in branch impedance due to the inclusion of 500kV as well as 10kV lines in one model or cases that have large *R/X* ratios. These are very interesting issues. Unfortunately, not a single genuine ill-conditioned case was found to be non-related to operating the system at the nose point in this research. This experience and the logical reasoning described below seem to indicate that all genuine ill-conditioned cases only occur near the maximum system load-ability point.

The Jacobian matrix is actually a sensitivity matrix. Although originated from the Newton method, the matrix has its own physical meaning and is independent of any solution methods. Since it is a sensitivity matrix, it can be used to investigate and quantify the degree of voltage sensitivity to system parameter variations. There are essentially two types of parameter variations to consider. One type is the variation of load levels and voltage settings. The other is the variation of impedance values. The first type variation can be analysed directly using the Jacobian matrix as follows:

$$\boldsymbol{J}\,\Delta\mathbf{x} = \Delta\mathbf{y} \tag{8.21}$$

where $\boldsymbol{J}$ is the Jacobian matrix. $\Delta\mathbf{x}$ and $\Delta\mathbf{y}$ are the voltage variation and load variation respectively. The second type variation can be dealt with as follows:

$$\boldsymbol{J}\Delta\mathbf{x} + \Delta\mathbf{z} = 0 \tag{8.22}$$

where $\Delta\mathbf{z}$ is the equivalent load variation caused by impedance changes. It can be seen that both types of parameter variations can be analysed using the ***J*** matrix. One can therefore conclude that the more fundamental definition of the ill-conditioning problem is essentially identical to the Jacobian-matrix condition number based definition. The later definition is also independent of the Newton method.

Due to the above reasoning, it becomes logically difficult to imagine 'other types' of ill-conditioned cases. If such cases did exist, it means that a large condition number would occur before the system reaching the nose point. According to the analysis of previous sections, the condition number would be even larger at the nose point. The situation, therefore, becomes an issue of how close to the nose point a case should be so that it can be characterised as ill-conditioned. From this perspective, it is possible that a case with 90% loading level could be considered as ill-conditioned while another case has to reach 98% loading level in order to qualify for the classification. The difference essentially rests on the slopes of their respective PV or QV curves and is a relative one. The experience on voltage stability analysis also supports this consideration: cases that indiscriminately include high voltage and low voltage branches can often experience 'local' voltage instability [53]. The Jacobian matrix becomes singular because of a localized area getting overloaded.

8.7 Conclusions

The studies described in this chapter comprehensively investigate the nature of ill-conditioned power flow problems from the unique perspective of voltage stability assessment. It is demonstrated that a large percentage of so-called ill-conditioned problems have nothing to do with the ill-conditioning of the Jacobian matrices. If a case is a genuine ill-conditioned case, it is essentially a voltage collapse case. The main conclusions of this work are summarized as follows:

- This work reveals, for the first time, that the so-called ill-conditioned power flow problems are either related to the non-existence of a power flow solution or caused by operating the system at the voltage collapse point. In other words, a genuine ill-conditioned power flow problem occurs only at the voltage collapse point, where the condition number of the Jacobian matrix becomes infinite. As a result, the well-established voltage stability assessment methods are the most effective tools to analyse ill-conditioned power flow problems.

- This work clarifies many questions related to the classical 'ill-conditioned' power flow cases. The findings suggest that it is unprofitable to research new algorithms dedicated to solving 'ill-conditioned' power flow problems. The problems are essentially voltage instability problems. The continuation method can provide a lot of information on the causes of ill-conditioning or power flow divergence, and is recommended for systems experiencing convergence problems.

Chapter 9

Conclusions

Deregulation has brought significant changes to the electric power industry. The need for and supply of system support services have become a very important issue for the new power market. There is a considerable need to find a technically sound method for the competitive procurement of reactive power support services. In response to the challenge, this thesis has investigated the problems of reactive power support needs of the generators or power producers. It has proposed a set of concepts and methods to evaluate the needs, which forms a component of the technical foundation for the management of reactive power support services. The main contributions of this thesis are as follows:

- The function and role of a generator's reactive power output have been investigated. It is revealed that a generator, though outputting reactive power, could still require reactive power support from the system in order to transmit its own active power. As a result, a generator may not always provide reactive power support to the system even if it has reactive power output. This finding brings a new perspective on the subject of how to compensate the reactive power support provided by generators.

- The concept of minimum reactive power need (Q_{min}) for generators has been introduced. It is defined as the least amount of reactive power needed from a generator to maintain the same degree of system security. A generator's reactive power output can be remunerated only if it is greater than Q_{min}. With the help of

Q_{min}, it becomes possible to determine which generator provides reactive power support to the system and by what amount. This concept makes it possible to analyze the problem quantitatively.

- A technically sound approach to determine the minimum reactive power needs of generators is proposed. Promising test results have been found from extensive case studies for actual large systems. These results have verified the validity and performance of the proposed concepts and method. The proposed method has thus become a tool to facilitate the establishment of price signals for the reactive power support services needed as well as offered by the generators.

- Based on the proposed concepts and methods, possible schemes for compensating reactive power support services are investigated. Two schemes are proposed in this thesis. One uses the minimum reactive power index to facilitate the allocation of system support costs. The other applies the index to determine the entitlement of a generator for reactive support payment. Both schemes have demonstrated the need to differentiate generators according to their locations and true reactive support levels.

- In addition, different methods for modeling a generator's reactive power limit have been evaluated by assessing their impact on power flow and system margin calculation. The study results suggest that although the PQ model is only an approximate model, it is sufficient for the determination of the minimum reactive power requirements.

- This research work has also solved an important puzzle of power flow analysis — the ill-conditioned power flow problems. It was found and proved mathematically that the ill-conditioned power flow occurs only at the voltage collapse point. In other words, the problem is essentially a voltage collapse problem and it is unprofitable to research new algorithms dedicated to solve 'ill-conditioned' power flow problems. A simple QV curve method is sufficient to diagnose the problem.

The problem identified in this research work has a significant implication to the profitability of power producers. It is possible that other methods to define and to calculate Q_{min} could be developed. The fact that a generator must produce certain amount of reactive power to facilitate its own active power transmission should not be denied. For the proposed method, more work is needed to fully explore its potential applications and limitations. Some of the future work includes:

- It is still very useful to test the proposed method on more actual systems. The objective is to discover and solve any practical and theoretical problems that may arise when using the methods for real-life applications. One example issue that needs to be addressed is how to deal with multiple operating scenarios. The solution efficiency of the method could also be improved. It may be possible to formulate the problem in one step and use an optimization method to find the Q_{min}.

- The proposed method has so far been focused on the heavy load conditions. Whether the proposed method can deal with the light load conditions is still not clear at present. During light load conditions, the concerns for system security may be different. The indices selected for establishing reference cases may also be different. Furthermore, the main interest in this case is the reactive power absorbing capability of the generators.

- With the above two problems addressed, it then becomes possible to establish a systematic procedure to support the competitive procurement of reactive power services from the generators. An ideal solution could be a set of price signals for the generators to decide if they want to generate their own reactive power or buy it from the system.

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Appendix A

Test Systems: BC Hydro System and Alberta System

A.1 BC Hydro System

The BC Hydro system is shown in Figure A.1. The 500 kV bulk transmission network connects the major generators in the northern and southern interior regions of the province with the major load centers in heavily populated southwest B.C. Electricity is supplied to the Lower Mainland and Vancouver Island from the Peace River hydroelectric system through Kelly Lake Substation and from the Columbia River system through Nicola Substation. BC Hydro is interconnected to Alberta by two 138 kV lines and one 500 kV line and to the United States by two 500 kV and two 230 kV lines.

A.2 Alberta Interconnected Electric System

The Alberta Interconnected Electric System (AIES) is shown in Figure A.2. Major generators are located in Areas 3, 9 and 14. They provide 82% active power generation to the system. In terms of loads, it is quite different from the BC Hydro system. Instead of only one load center, loads disperse over the system and many load centers exist. Major load centers are Areas 6, 3, 8, 9, 5 and 14. Almost 70% load is in these areas. Table A.1 lists the area numbers and names of the AIES system.

The test cases of both BC Hydro system and Alberta system are summarized in Table A.2.

Appendix A

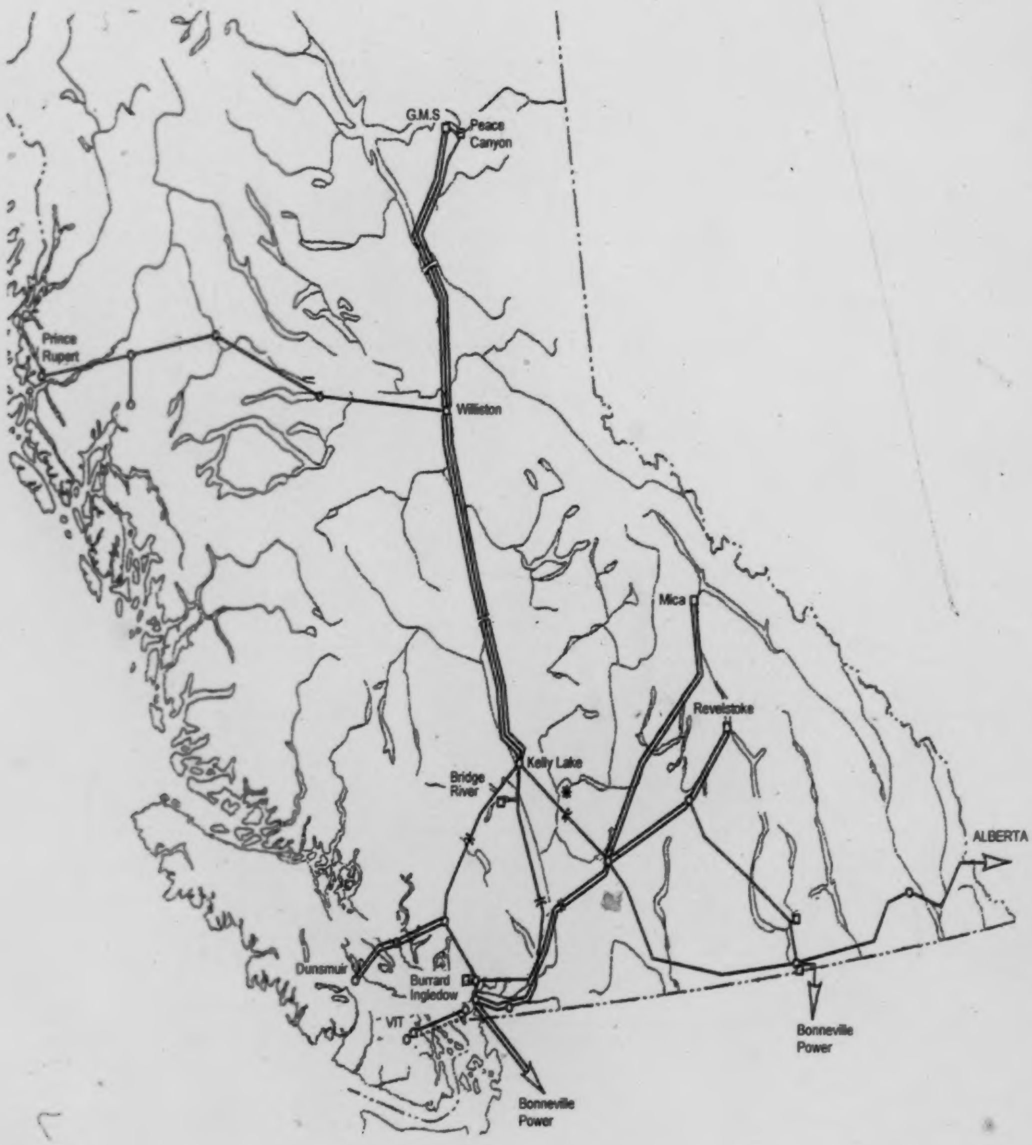

Figure A.1: BC Hydro system

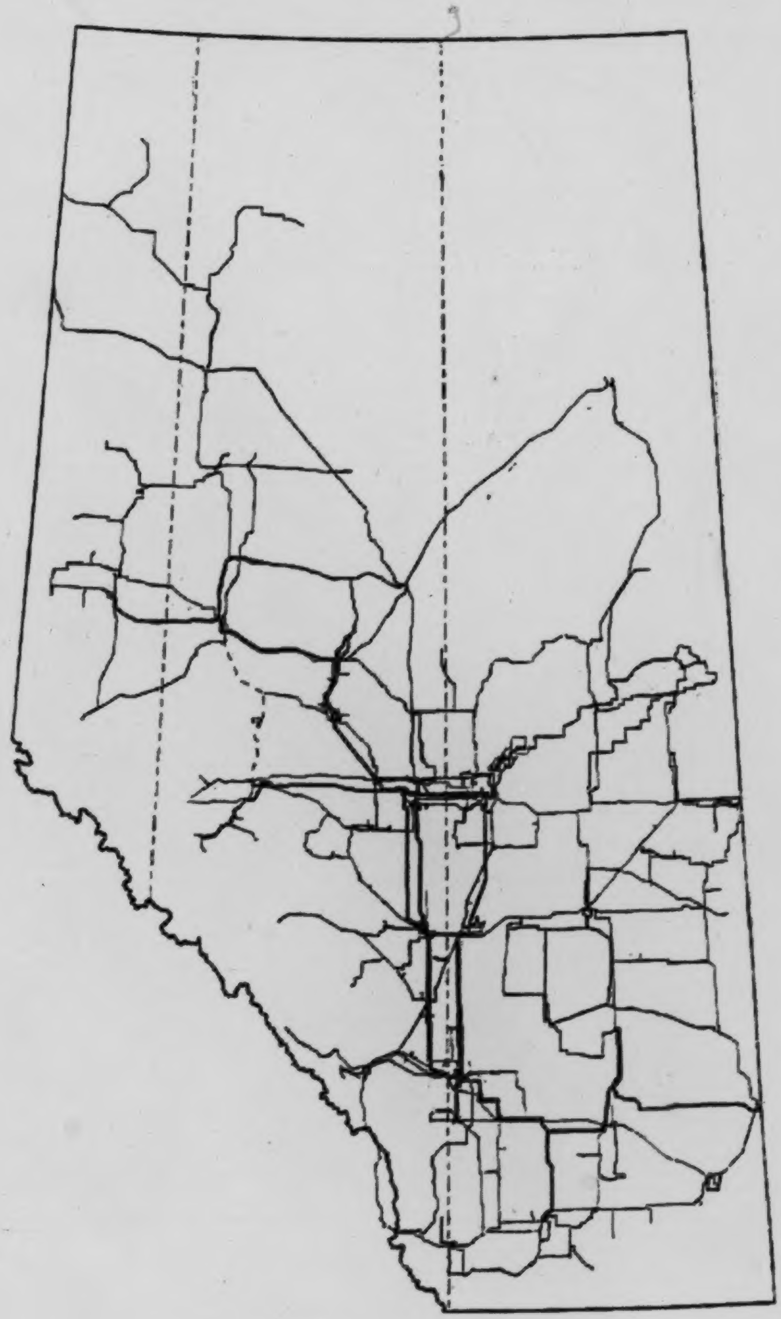

Figure A.2: Alberta system

Table A.1: Area number and names of the Alberta system

Area No.	Area Name	Geographical Name	Main Areas
1	'E. CENT.'	East Center	Camrose, Provost, Wainwright
2	'W. CENT.'	West Center	Fox Creek, Judy Creek, Edson
3	'N. CENT.'	North Center	Westlock, Barrhead, Sundance, Redwater, Pow Ch…
4	'MED. HAT'	Medicine Hat	Medicine Hat
5	'S. CENT.'	South Center	Red Deer, Wetaskiwin
6	'CALGARY '	Calgary	Calgary
7	'BOW VAL.'	Bow Valley	Airdrie, Canmore
8	'S. ALTA.'	South Alberta	Lethbridge, Langdon
9	'EDMONTON'	Edmonton	Edmonton
10	'PEACE RV'	Peace River	Seal Lake, Peace River, Wesley Creek, Hotchkiss, Zama Lake
11	'GRNDE PR'	Grande Prairie	Hines Creek, Kistuan River, Elmworth, Bezanson
12	'SLAVE LK'	Slave Lake	Wabasca, Mitsue, Cranberry, Sturgeon
13	'BON/LLYD'	Lloydminster	Ethel Lake, Whitefish, Vermillion, Lloydminster
14	'DRUMHELL'	Drumheller	Drumheller, Sheerness, Youngstown, Bigfoot, Bigknife, Battle River
15	'BC/WSCC '	BC/WSCC	BC/WSCC
25	'FT MCMRY'	Fort McMurry	Fort McMurry, Parsons Creek, Ruth Lake, Hangingstone
26	'SK-TIE '	Sask. Tie	McNeill (Tieline to Sask.)
30	'CAPTIVE '		Parsons Creek, Ruth Lake, Hangingstone, Syncrude, Daishowa
35			1201L (Tieline to BC)
89	'KANELK '		Banff
90	'P & G '	P & G	P&G plant
91	'SYNCRUDE'	SYNCRUDE	Syncrude
92	'SUNCOR '	Suncor	Suncor
93	'DIASHOWA'	DIASHOWA	Daishowa plant
97			St. Mary, Belly River

Table A.2: Summary of the two test systems

Case Information	BC Hydro System	Alberta System
No. of Buses	1044	1755
No. of Branches	1426	1956
No. of Plants	108	114
No. of Machines	120	157
No. of Loads	374	1097
No. of Transformers	625	1019
Total Generation (Active)	9242MW	7396MW
Total Generation (Reactive)	1383Mvar	1670MVar
Total Load (Active)	8689MW	6827MW
Total Load (Reactive)	2596MVar	2000MVar

Appendix B

Data of Sample Test Systems

Besides the actual BC Hydro and Alberta systems, a number of sample test systems are used in this thesis. The purpose of these simplified test systems is to illustrate a phenomenon, to introduce a concept or to analyse a problem. The data of these test systems are documented in this Appendix.

In most of the sample test systems, line susceptances, also known as line chargings or line shunts, are not explicitly expressed. Their impact is included by means of Thevinin equivalent circuit. The line resistances are neglected in most test cases because they are much smaller than line reactances, i.e. $R<<X$, especially for high voltage transmission lines. When per unit is used in a case, the base power is 100MVA and the base voltage is the nominal voltage of the corresponding voltage level.

B.1 Sample System in Figure 2.4

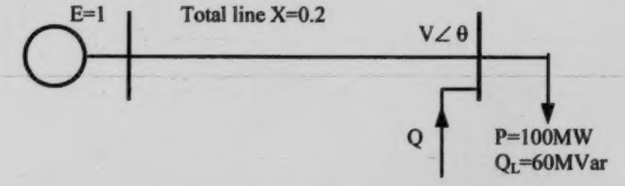

Table B.1: Data of the sample system in Figure 2.4

Line parameters		Operating parameters	
X	0 ~ 0.2 pu	E	1.0 pu
R	0	P	100MW
		Q_L	60MVar
		Q	0~60MVar

B.2 Three-Bus System in Figure 3.3

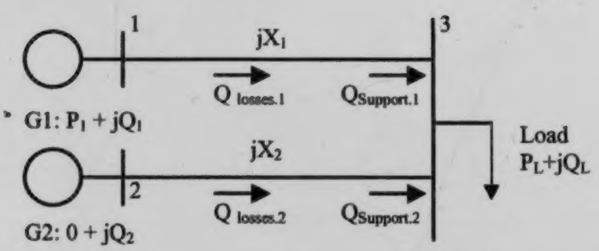

Table B.2: Data of the three-bus system in Figure 3.3

Line parameters		Operating parameters used in Section 3.3-1)		Operating parameters used in Section 3.3-2)	
X_1	0.2 pu	E_1	1.0 ~ 0.7 pu	E_1	1.0 pu
X_2	0.2 pu	E_2	1.0 pu	E_2	1.0 pu
All R	0	P_L	80MVar	P_L	0 ~ 260MW
		Q_L	100MVar	Q_L	100MVar
		P_1	80MW	P_1	0 ~ 260MW
		P_2	0	P_2	0

B.3 Five-Bus System in Figure 3.7

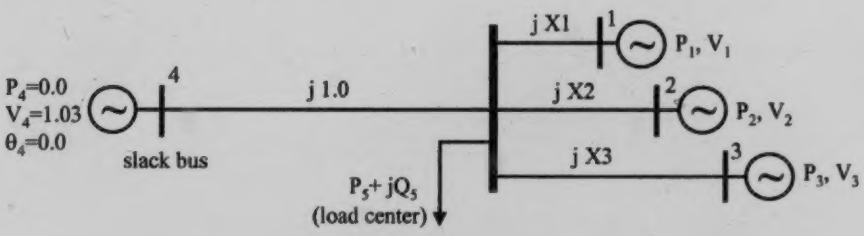

Table B.3: Data of the five-bus system in Figure 3.7 (Case 1)

Line parameters		Operating parameters			
X_1	0.1 pu	V_1	1.0 pu	P_1	60MW
X_2	0.2 pu	V_2	1.0 pu	P_2	60MW
X_3	0.3 pu	V_3	1.0 pu	P_3	60MW
All R	0	V_4	1.03 pu	P_5	180MW
				Q_5	60MVar

Table B.4: Data of the five-bus system in Figure 3.7 (Case 2)

Line parameters		Operating parameters			
X_1	0.1 pu	V_1	1.0 pu	P_1	30MW
X_2	0.1 pu	V_2	1.0 pu	P_2	60MW
X_3	0.1 pu	V_3	1.0 pu	P_3	90MW
All R	0	V_4	1.03 pu	P_5	180MW
				Q_5	60MVar

Table B.5: Data of the five-bus system in Figure 3.7 (Case 3)

Line parameters		Operating parameters			
X_1	0.1 pu	V_1	1.02 pu	P_1	60MW
X_2	0.1 pu	V_2	1.01 pu	P_2	60MW
X_3	0.1 pu	V_3	1.0 pu	P_3	60MW
All R	0	V_4	1.03 pu	P_5	180MW
				Q_5	60MVar

B.4 Single Generator – System Test System in Figure 4.1

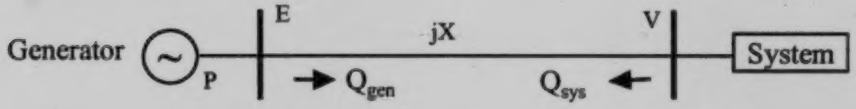

Table B.6: Data of the single generator-system test system in Figure 4.1

Line parameters		Operating parameters	
X	0.1 pu	E	0.9 ~ 1.1 pu
R	0	V	1.0 pu
		P	300MW